Bounding Distributed Energy Balancing Schemes for WSNs via Modular Subgames

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***Abstract*—This paper studies means of archiving energy balance in Wireless Sensor Networks (WSNs) as criterion to increase network lifetime. While protocols for balancing energy through centralized computation have been established, effective solutions for more realistic distributed settings are still an open research field. In this work, we present a novel approach for balancing energy in WSNs in a distributed manner by solving a sequence of modular strategic subgames. In addition, we prove theoretical bounds for its solution quality. To the best of our knowledge, we are the first to explicitly provide theoretical bounds for distributed WSN energy balancing schemes. We do so by formulating two specific routing games in which sensor nodes act as strategic agents with interests in energy balance. The first formulated game is a monotone utility game and by exploiting and adapting some existing results, we can prove a meaningful relative bound for its solution value via our distributed modular subgames scheme under realistic conditions. For the second game we provide an absolute bound. We round up our theoretical results by an experimental evaluation, where our modular subgames scheme shows increased performance compared to state-of-the-art algorithms.**

I. Introduction

We study Wireless Sensor Networks (WSNs) which due to their practical applications, including (forest) fire detection, traffic surveillance, climate monitoring, tracking (location sensing), and more, have gained particular attention in recent years. WSNs are composed of multiple sensor nodes whose sensed data are collected at one or more data gathering sink(s). The nodes usually have limited resources, i.e., small battery, limited memory, limited computing power. Energy consumption when forwarding data is a key local criterion and balanced energy consumption among the nodes composing the network prolongs network lifetime. A well known radio transmission property is that small distance transmission consumes less energy than direct transmission to distant nodes. Instead of sending their data directly to the sink, nodes may shift to multi-hop routing to regulate energy consumption.

In the following, we will present a novel approach for balancing energy in WSNs in a distributed manner by solving a sequence of modular strategic subgames. We will prove solution quality bounds and present an experimental evaluation.

A. Related Work

From 2004 to 2011, the groups around Leone, Nikoletseas, and Rolim established a series of fundamentals regarding energy balance in WSNs (i.a., [1], [2], [3]). They provided theoretical analyses and experimental results for specific static network models, centralized offline data propagation schemes and first distributed online schemes. In the years 2011 and 2012, Nikoletseas et al. extended this line of research. In 2011, they introduced a nodes mobility model and a new distributed online energy balancing protocol (without theoretical or experimental results) [4] and later studied heterogeneous node placements and presented improved performance in experimental tests of a new distributed algorithm compared to [1] (if nodes have extended knowledge about the network density and if the number of messages is constant over time and equal for all nodes) [5]. The distributed schemes in [3] and [5] will serve as benchmark algorithms in the simulation section of this paper. Further related research and recent developments regarding WSN energy balancing algorithms include: studies of lifetime maximization by focus on a suitable node distribution avoiding “energy holes” (e.g., [6]); experimental evaluations of energy balance protocols for random sensor network topologies considering coverage and connectivity problems (e.g., [7]); experimental evaluations of explicitly integrating energy balancing ideas in standard clustering data propagation protocols as LEACH (e.g., [8]); and an analytical study of energy balance bounds of two-hop based mixed data transmission (as even in energy-balance optimal topologies an energy-balanced flow may not exist at all) [9]. Other than the later work, we will study bounds of balancing schemes not on the topology. First game-theoretic models to find energy balancing solutions for WSNs can be found in [10] and [11], where schemes for static offline, respectively, cluster full knowledge settings are presented.

B. Contribution of this Work

In contrary to former work, we will not only provide a distributed energy balancing scheme for WSNs based on a game-theoretic model, we will furthermore exploit existing results from algorithmic game theory to prove theoretical solution quality bounds. To the best of our knowledge, we are the first to explicitly introduce theoretical solution bounds for distributed energy balancing schemes for WSNs. We round up our theoretical results by an experimental evaluation. As will be demonstrated, not only can we prove theoretical bounds,

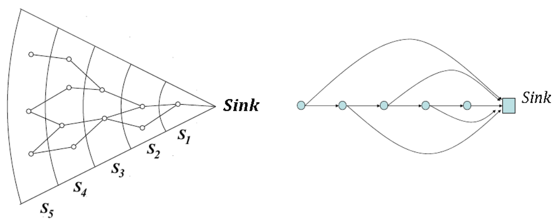


Fig. 1. An energy-balance optimal topology on the left, sensors transmit data to the next slice or directly to the sink (not represented). A simple model of it on the right.

our new distributed energy balancing scheme for WSNs also outperforms the considered state-of-the-art algorithms.

II. Data Gathering Model

We use the following data gathering network definition (modified from its version in [3]) as preliminary WSN model.

Definition 1 (Data Gathering Network): A data gathering network is defined to be the directed acyclic simple graph $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$. $V = \{0, .., n\}$ is the node set of G with designated data gathering sink 0. The other nodes in V represent sensor nodes. E is the set of directed edges in G. We assume that $G \setminus \{i = 0\}$ can be decomposed into slices $S_1, S_2, ..., S_m$ such that edges only exist from one slice to the next lower one and every node has at least one edge towards the next lower slice. In addition, we assume that every node $i = 1, ..., n$ has a direct edge to the sink 0. (Note that graph G is multiply connected, i.e., for every node in a slice higher than S_1 there exist at least two paths to the data gathering sink). An example of the resulting topology is taken from [3] and can be found in Fig. 1. For all $i = 1, ..., n$, let c_{i0} denote the costs of edge $(i, 0)$ and let c_i denote the costs of any other outgoing edge of node i (we will slightly abuse the notation). Costs represent energy that has to be spent by node i for sending one unit of flow along the edge. We assume that c_i has equal value for all $i = 1, ..., n$ and $c_{i0} > c_i$ for all $i = 2, ..., n$ (and $c_i = c_{i0}$ for $i = 1$). Furthermore, for all nodes i, j in the same slice let $c_{i0} = c_{j0}$ and let $c_{i0} < c_{j0}$ if $i \in S_{l_1}$ and $j \in S_{l_2}$ and $l_1 < l_2$ (i.e., the costs for sending flow directly to the sink are the same for all nodes in the same slice and increase as the distance of the slice to the sink increases). Scalar d refers to the total flow value that is supposed to be sent through the network to the sink. For $i = 1, ..., n$, parameter g_i denotes the fraction of flow value generated by node i and b_i is the initial energy amount available to i.

We briefly repeat the notion of an *energy-balanced flow* and *network lifetime* taken from [3] and [2].

A. Network Lifetime & Energy Balance

Given data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$, we want to route the given total flow (value) d along edges such that each node $i = 1, ..., n$ generates $g_i \cdot d$ units of the total flow, the flow on the edges is non-negative ($f_e \geq 0$, $\forall e \in E$), and flow conservation is satisfied, i.e.,

$$g_i d + \sum_{e \in \delta^-(i)} f_e = \sum_{e \in \delta^+(i)} f_e, \ \forall i = 1, ..., n.$$

Let $\{f_e\}$ be a non-negative multiple-source single-sink flow in G satisfying above conditions.

Definition 2 (Lifetime of G w.r.t. $\{f_e\}$): The lifetime of network G w.r.t. $\{f_e\}$ is the minimum lifetime of the nodes composing the network w.r.t. $\{f_e\}$, i.e.,

$$\min_{i=1,\dots,n} \frac{b_i}{\sum_{e \in \delta^+(i)} f_e c_e} \ .$$

Definition 3 (Energy-Balanced Flow in G): Flow $\{f_e\}$ is called energy-balanced in G if

$$\exists k : \forall i = 1, ..., n \ \sum_{e \in \delta^+(i)} f_e c_e = k b_i \ .$$

The topology of our data gathering network and the settings for transmission costs along an edge ensure that every energy-balanced flow maximizes the lifetime of the network (see [3]). Our data gathering network is therefore also called an *energy-balance optimal network*. Nevertheless, an energy-balanced flow might not exist at all (see e.g., [2]).

B. Centralized Offline Computation of Energy Balance

The centralized offline problem of finding a flow that maximizes lifetime in our data gathering network can be solved by solving the linear program (LP) of Problem 2 in [3]. Instead of searching for a maximum lifetime flow, we will focus on the computation of an energy-balanced flow with prospects to be found much easier in a distributed manner. We will present a centralized offline approach that computes an energy-balanced flow if existent and a best approximation of energy balance if it does not exist. The solution will later on serve as upper bound for the distributed solutions. For this purpose, we introduce *balance-factors* for the nodes.

Definition 4 (Balance-Factor of a Node in G w.r.t. $\{f_e\}$): Given data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$, $\forall i \in V \setminus \{0\}$ the *balance-factor* of node i w.r.t. $\{f_e\}$ is

$$k_i = \frac{\sum_{e \in \delta^+(i)} f_e c_e}{b_i} \ .$$

Note that a flow $\{f_e\}$ is energy-balanced iff the k_i are equal for all $i = 1, ..., n$.

Definition 5 (Best-Energy-Balanced Flow): A flow $\{f_e\}$ is called *best-energy-balanced* in $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ iff it is an optimal solution of the following *Centralized Offline Balance (CentralEBOpt)* LP:

$$\min_{\beta, \{k_i\}, \{f_e\}} \quad \beta \qquad (1a)$$

$$k_i - k_j \leq \beta \qquad \forall\, i, j = 1, ..., n \quad (1b)$$

$$k_i b_i = \sum_{e \in \delta^+(i)} f_e c_e \qquad \forall\, i = 1, ..., n \quad (1c)$$

$$g_i d + \sum_{e \in \delta^-(i)} f_e = \sum_{e \in \delta^+(i)} f_e \quad \forall\, i = 1, ..., n \quad (1d)$$

$$f_e \geq 0 \qquad \forall\, e \in E \quad (1e)$$

Note that above LP is bounded and that there always exists a feasible solution (i.e., any routing of the demands through the network to the sink, as we do not have limiting capacities on the edges), hence there always exists an optimal solution.

Proposition 1: If an energy-balanced flow in G exists then every best-energy-balanced flow is energy-balanced and vice versa.

Proof: For an energy-balanced flow, there exists k such that $\forall i = 1, ..., n \;\; \sum_{e \in \delta^+(i)} f_e c_e = k b_i$. When inserting this solution (setting all $k_i = k$) in LP (1), we get a solution value of $\beta = 0$ which is optimal. Now given a best-energy-balanced solution $(\beta, \{k_i\}, \{f_e\})$, assume that $\{f_e\}$ is not an energy-balanced flow while another energy-balanced flow $\{f'_e\}$ exists. Then the existing energy-balanced flow $\{f'_e\}$ is also a solution to LP (1) with a better solution value as all k'_i are equal. This is a contradiction to $(\beta, \{k_i\}, \{f_e\})$ being optimal. ■

III. Decentralized Solutions for Energy Balancing Routing Games

The task of balancing energy in a wireless sensor network can be viewed as a global problem that has to be tackled in a decentralized manner due to resource and communication limitations. We will apply methods and tools of game theory, here autonomous agents make local decisions resulting in global outcomes. In our static data gathering model, we will in the following assume that nodes do not only decide to which node they will forward their message next, but they will choose a complete path towards the sink using one- or multi-hop routing. We will use this model to provide a theoretical basis for bounding distributed WSN energy balancing protocols also for model variations. In a follow-up work we want to adapt the model such that it is also suitable for WSNs with mobile nodes and for networks with changing connectivity.

Before, we describe our distributed solution approach and prove bounds, we build two game models. While the first one employs less network information, its social optimum is only close to optimal energy balance. In the second provided game, the social optimum equals energy balance, but more information about past activities is needed. Trade-offs will be discussed in the course of this paper.

A. Classical Routing Game

We define a special maximization version of a selfish routing game that we call *classical WSN routing game*.

Definition 6 (Classical WSN Routing Game): Given data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ with designated sink $0 \in V$, a *classical WSN routing game* is defined as the Wardrop game (often also referred to as non-atomic routing game) with set of players $N = V \setminus 0 = \{1, ..., n\}$ in which each player i aims to route its initial demand $d_i = g_i \cdot d$ as profitable as possible towards the sink along existing $(i, 0)$-paths in G. The set of $(i, 0)$-paths is denoted by $\mathcal{P}_i$ and let $\mathcal{P} = \cup_i \mathcal{P}_i$. An outcome or feasible flow of the game is a function $\mathbf{f} : \mathcal{P} \to \mathbb{R}_{\geq 0}$ routing all demand of the players to the sink. We set $f_e = \sum_{\mathbf{p} \in \mathcal{P} : e \in \mathbf{p}} f_{\mathbf{p}}$ and each edge has load-dependent costs $l_e(\mathbf{f}) = l_e \cdot f_e$ with $l_e = \frac{c_e}{b_{i : e \in \delta^+(i)}}$. (The latency factor l_e corresponds to the energy amount for forwarding one unit of flow along the edge divided by the initially available energy amount of the tale node i of edge e). A player receives a reward r_{p_i} for every unit of flow sent along a path $p_i \in \mathcal{P}_i$. We slightly abuse the notation and denote the amount of flow sent by player i along path p_i with f_{p_i}. We set r_{p_i} to be equal for all $p_i \in \mathcal{P}_j, \forall i, \forall j$, and assume $r_{p_i} \gg \sum_{e \in p_i} l_e(\mathbf{f})$ for every possible outcome. The utility function of a player i is defined as

$$u_i(\mathbf{f}) = \sum_{p_i \in \mathcal{P}_i} \left(r_{p_i} - \sum_{e \in p_i} l_e(\mathbf{f}) \right) f_{p_i}.$$

In the classical WSN routing game, every player has an incentive to send its own flow as cheap as possible while receiving a reward for every flow unit sent. Due to the classical routing game structure, a player considers that own costs will depend on the other players' abilities to minimize energy consumption, too. Note that while we have equilibrium existence due to the linear, hence continuous, and monotone growing cost function, this will play a minor role for the distributed solution approach of the classical WSN routing game. Let us now formally define the social welfare function for our game and prove some further nice properties, which will help us build a bounded decentralized solution scheme in the next subsection III-C.

Definition 7 (Social Welfare): The social welfare of a subset $\mathbf{f}$ of an outcome of a classical WSN routing game is

$$V(\mathbf{f}) = \sum_{i \in N} u_i(\mathbf{f}) = \sum_i \sum_{p_i \in \mathcal{P}_i} \left(r_{p_i} - \sum_{e \in p_i} l_e(\mathbf{f}) \right) f_{p_i}.$$

A socially optimal flow in our classical WSN routing game yields a feasible flow with minimum total energy consumption under the assumption that players act selfish and routing costs are load dependent. While this objective is not equivalent to maximizing global energy balance, it favors energy balance among nodes sharing the same paths. We will later on provide experimental tests to make results comparable to the actual optimal best-energy-balanced solution in the according WSN. Note that the social optimum of the classical WSN routing game can be computed by solving a quadratic program with positive definite matrix in the objective function and linear constraints.

The following Lemma proves that a classical WSN routing game is a monotone utility game which guaranties properties which we will exploit later on.

Lemma 1 (Monotone Utility Game Properties): The classical WSN routing game is a monotone utility game.

Proof: For the social welfare function V of a balanced WSN routing game holds:

1) V is submodular (when restricted to a suitable fine discretization of the routed flow), i.e., for any $\mathbf{f} \subset \mathbf{f}' \subset \mathbf{f}''$ and any element $f \in \mathbf{f}''$: $V(\mathbf{f} + f) - V(\mathbf{f}) \geq V(\mathbf{f}' + f) - V(\mathbf{f}')$. I.e., the marginal benefit to social welfare of adding new flow diminishes as more flow is added.
2) The total value for the players is less than or equal to the total social value: $\sum_{i \in N} u_i(\mathbf{f}) \leq V(\mathbf{f})$.
3) The value for a player is at least her/his added value for the society: $u_i(\mathbf{f}) \geq V(\mathbf{f}) - V(\mathbf{f} - f_i)$.

Hence, the classical WSN routing game is a utility game. The classical WSN routing game furthermore has a monotone growing social welfare function, i.e., for all $\mathbf{f} \subseteq \mathbf{f}'$, $V(\mathbf{f}) \leq V(\mathbf{f}')$. (The more flow is added the higher the value for society due to increased number of rewards). ■

Note that utility games, formally introduced by Adrian Vetta [12], are defined as games with submodular social welfare function aimed to be maximized. As done for general selfish routing games in [12], we defined a maximization version of our specific routing game in order to later on exploit some of the results for utility games.

In the following, we present an alternative game formulation in which players' costs do not only favor balanced energy consumption, but energy balance is more directly addressed. Nevertheless, for this game we cannot prove the same property of being a monotone utility game.

B. Balanced Routing Game

We modify the game from the previous section such that edge costs reflect the actual energy consumption at this point of the network and total costs are split between the players via special player cost functions.

Definition 8 (Balanced WSN Routing Game): Given data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ with designated sink $0 \in V$. The *balanced WSN routing game* is the strategic game (N, P, C) with player set N corresponding to the set of nodes $V \smallsetminus 0$ and set $P = \cup_i P_i$ being the product of pure strategy sets P_i of players $i \in N$. The set P_i corresponds to the set of all $(i, 0)$-paths in G. The vector $\mathbf{p} = (p_1, ..., p_n) \in P$ denotes an outcome of the game, $d_i = d \cdot g_i$ denotes the initial demand of player i, and $f_e(\mathbf{p}) = \sum_{i \in N} ((p_i \cap e) d_i)$ denotes the amount of flow on edge e in outcome $\mathbf{p}$. General costs for using edge e in $\mathbf{p}$ are given by $\frac{c_e}{b_{i:e \in \delta^+(i)}} \cdot f_e(\mathbf{p})$. In addition, a unique cost function $C : P \to \mathbb{R}_{\geq 0}$ is defined as

$$C(\mathbf{p}) = \max_{i,j \in N} \frac{1}{n} \Big(\sum_{e \in \delta^-(i)} \frac{c_e}{b_{i:e \in \delta^+(i)}} f_e(\mathbf{p}) - \sum_{e \in \delta^-(j)} \frac{c_e}{b_{j:e \in \delta^-(j)}} f_e(\mathbf{p}) \Big) . \quad (2)$$

The value of the cost function corresponds to $\frac{1}{n}$ times the maximum difference of individual energy consumption of nodes in outcome $\mathbf{p}$. The cost function of player $i \in N$ is given by $u_i : P \to \mathbb{R}_{\geq 0}$. The cost functions of all players are equal, we have $u_1 = u_2 = ... = u_n = C$.

The above game is explicitly defined for pure strategy game play. The introduction of mixed strategies (i.e., probability vectors over the pure strategies) is straight forward when needed. The defined game models the interaction of autonomous agents whose goal is to minimize the social costs while each being limited by the strategies available to her/him. We emphasize that this game is a potential game with a potential function being equal to the social costs function (i.e., the sum of costs over all players). In other words, there exists a unique function describing the incentives for every player to change her/his strategy (see e.g., [13]). As the potential function equals the social costs function, the social value of a best Nash equilibrium equals the social optimum of this game (see e.g., [14]). (A mixed Nash equilibrium exists as we are considering a finite strategic game due to Nash's Theorem). A best Nash solution can be computed by solving the centralized energy balance LP (1) from section II-B. This is obvious when comparing the objective function of LP (1) to the social cost function (which are equal).

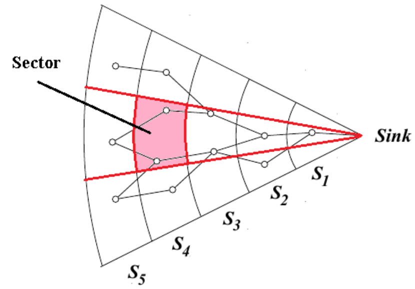


Fig. 2. Sketch of an energy-balance optimal topology. Network slices are indicated in vertical distance from the sink. Each slice is split into sectors of equal size within a slice.

C. Decentralized Modular Subgames Scheme

We now present our decentralized modular subgames solution approach for the presented games and prove solution quality bounds.

1) Modular Subgames Generation & Scheme: The core of the distributed modular subgames solution concept is the generation of disjoint subgames. We propose two kinds of suitable partitionings of the nodes to form subgames which correspond to realistic groupings of neighboring nodes.

Definition 9 (Feasible Subgames Partitioning): A *feasible subgames partitioning* of $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ is a set partitioning P of the nodes V such that for every two nodes in the same set Q of the partitioning, there exists a connecting path in the induced subgraph $E(Q)$. The nodes of each set of the partitioning form the players of a subgame of a (global) game defined on G with strategies and costs/utilities as in the game.

Definition 10 (Homogeneous Feasible Subgames Partitioning): A feasible subgames partitioning of G is called *homogeneous*, if the total number of nodes in a subgame is equal for all subgames.

Finding a partitioning of nodes as described in Definition 10 is in general known to be a NP-hard problem (see e.g., [15]). But going the other way around, when needed, the following network construction yields a homogeneous feasible subgames partitioning.

As described in section II, we are considering energy-balanced optimal network topologies in which nodes are placed in different slices. We pick up an idea from [5] and in addition cut each slice into sectors of equal size per slice. A sketch of such a partitioning into sectors is given in Fig. 2. We place nodes in the network such that each sector of the network contains the same number of nodes (this is not displayed in the figure). Furthermore, we assume that every node of a sector is connected to every node in the parallel sectors of the neighboring slices. A motivation for such a node placement could for example be network coverage. An improved partitioning into sectors and distribution of nodes throughout the network is possible as long as the number of contained nodes is equal for all sectors and connectivity is satisfied. Now regarding the actual subgames generation, we employ a given non-negative integer parameter h. Assuming that the total number of slices is a multiple of $h+1$, we cluster parallel sectors of $h+1$ slices together to form a subgame starting from the most inner slice. This way all nodes may coordinate their actions with nodes in the same sector and nodes in sectors that are at most h hops away. Applying this rule under given assumptions, we can generate disjoint subgames with same number of nodes. Given a feasible subgames partitioning, our modular subgames solution approach is as follows:

Definition 11 (Modular Subgames Scheme): Given

- data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$,
- classical or balanced WSN routing game of G,
- a feasible subgames partitioning P of G,
- an order of the subgames of P,

the players of the first subgame, according the given order, play best response with respect to the sum of outcomes over the players in the current subgame. Next the players of the second subgame play best response with respect to sum of outcomes over the players in the current subgame to the strategies chosen by the players from the previous subgames. Players of the previous subgame cannot change their chosen strategies. After this, players of the third subgame play best response to past moves and so on until all subgames have been played.

Instead of playing one subgame after the other, in the classical WSN routing game, subgames with disjoint strategy sets (i.e., disjoint $(i, 0)$-paths of nodes i that are not in the same subgame) may be played simultaneously as long as the subgames scheduled in between them have also strategy sets that are disjoint from them and each other.

2) An Absolute Bound: For the modular subgames scheme on the balanced WSN routing game, we can prove an absolute bound for energy balance, which is derived from the maximum energy consumption in the solutions and depends on the order of playing the subgames. In the following, we will slightly abuse the notation and let c_q denote the highest costs for sending one unit of flow from slice S_q directly to the sink.

Theorem 1: Given data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ and a feasible subgames partitioning. Let m be the total number of slices in G. Recall that b_i denotes the initially available amount of energy and $d_i = g_i d$ is the demand of node $i \in V \setminus 0$. For each subgame s, let j_s indicate the highest slice index of a slice it covers. When the modular subgames scheme is applied to solve the balanced WSN routing game of G such that subgames are played in an order where subgames with smallest j_s are played first, than subgames with second smallest j_s and so on until all games have been played, then the maximum energy consumption deviation between two nodes composing the network is at most $\max_i d_i \frac{c_m}{b_i}$.

Proof: For simplicity reasons, let us consider the case where all nodes have same available energy amount $b_i = 1$. Recall that we solve a subgame by letting the players play best response with respect to the sum of the costs for players in the subgame. Consider the first subgames covering slices $S_1, ..., S_{q_1}$. A feasible solution is a solution where all nodes send their data directly to the sink with energy consumption $c_{i0} d_i$ and $c_{i0} \leq c_{q_1}$ and $d_i \leq \max_j d_j$. Hence, the maximum energy consumption (and therefore also the maximum energy deviation) is bounded by $c_{q_1} \cdot \max_j d_j$. When starting to play the next disjoint subgames covering slices up to S_{q_2}, all strategies from the former played subgames are fixed, they cannot be changed. We have $c_{q_1} < c_{q_2}$ by construction of the network. If a node from a next scheduled subgame sends demand directly to the sink, its energy consumption is bounded by $c_{q_2} \cdot \max_j d_j$ and energy consumption of nodes in lower slices is not affected. Due to the player cost functions, the nodes of the current subgame will only decrease their current energy consumption by choosing a multi-hop route while increasing the energy consumption of other nodes up to a point where their energy consumption is equal. Hence, the resulting energy consumption for every node is bounded by $c_{q_2} \cdot \max_j d_j$. For further subgames to be played, we continue with the same arguments until all m slices have been completely covered. Hence, energy consumption for every node is bounded by $c_m \cdot \max_j d_j$. ∎

The proven bound may seem trivial at first glance, but will prove to be a relatively good bound for distributed energy balancing schemes in the experimental tests. Although this type of scheduling may delay the arrival of messages of nodes far away from the sink, the delays here are calculable and can be reduced in dependence of the abilities of the nodes. Assuming that nodes are able to communicate within a subgame, the more slices are covered by a subgame, the lower the latency. By these means, for the first time to our knowledge, we give an explicit bound for an energy balancing scheme which also shows best performances in experimental test in comparison to state-of-the-art algorithms as will be presented in section IV.

3) Bounding the Price of Modularity: For the classical WSN routing game, we can exploit its properties as monotone utility game to prove a relative bound for the modular subgames scheme. Inspired by the famous price of anarchy which describes the ratio of the worst Nash equilibrium and the social optimum, we call the chosen quality criterion *price of modularity*. The basis for our proof gives the following solution concept and bound by Mirrokni and Vetta [16].

Theorem 2 (Thm. 3 from [16]): Consider an arbitrary solution in a monotone utility game. Suppose that each time step, we select a player at random and make a best response move for that player. The social value of the solution is at least $\frac{1}{2n}$ times the maximum possible social value (n being the number of players).

Extending ideas from above theorem, we will derive and prove a bound for our modular subgames solution concepts which integrates the ability of nodes to communicate and coordinate themselves within their local vicinity.

Theorem 3 (Bounding the Price of Modularity): Given classical WSN routing game of data gathering network $G = (V, E, d, \mathbf{g}, \mathbf{b}, \mathbf{c})$ and a homogeneous feasible subgames partitioning such that each subgame has the same amount of players ϑ. The solution via the modular subgames scheme with a random order of the subgames is at least $\frac{\vartheta}{2n}$ times the social optimum.

Proof: What needs to be shown, is that (1) the super game of the classical WSN routing game in which a super player represents the players of a subgame is again a monotone utility game and (2) that the values of the social optimum in the super game and in the initial game are equal.

We start with proving (2): In the super game and in the initial game, we have exactly the same $(i, 0)$-paths and same demands d_i for every node $i \in V$. Furthermore, the cost functions on the edges as well as the rewards for routing one unit of flow to the sink are equally defined. In the social optimum of the classical WSN routing game, routes and demands are chosen such as to maximize the sum of the utilities for all demand routed to the sink. If we cluster $(i, 0)$-paths of several players to assign them as strategies to a super player, the social optimum does not change.

To prove (1), we show that the super game of classical WSN routing game is a monotone utility game: Let q be the total number of subgames $j = 1, ..., q$ with player sets N^j of the given homogeneous feasible subgames partitioning P. We slightly abuse the notation and denote the according super player of subgame j also with j. (This should not lead to any confusions in the following). The utility function of a super players j is defined as

$$u_j^P(\mathbf{f}) = \sum_{i \in N^j} u_i(\mathbf{f}) = \sum_{i \in N^j} \sum_{p_i \in \mathcal{P}_i} \left(r_{p_i} - \sum_{e \in p_i} l_e(\mathbf{f}) \right) f_{p_i}.$$

The social welfare of a subset $\mathbf{f}$ of an outcome of the super game is defined as

$$V^P(\mathbf{f}) = \sum_{j=1}^{q} \sum_{i \in N^j} u_i(\mathbf{f}).$$

Following holds:

1) V^P is submodular (when restricted to a suitable fine discretization of routed flow), i.e., for any $\mathbf{f} \subset \mathbf{f}' \subset \mathbf{f}''$ and any $f \in \mathbf{f}''$: $V^P(\mathbf{f} + f) - V^P(\mathbf{f}) \geq V^P(\mathbf{f}' + f) - V^P(\mathbf{f}')$. I.e., the marginal benefit to social welfare of adding new flow diminishes as more flow is added.
2) The total value for the players is less than or equal to the total social value: $\sum_{j=1}^{q} u_j(\mathbf{f}) \leq V^P(\mathbf{f})$.
3) The value for a player is at least her/his added value for the society: $u_j(\mathbf{f}) \geq V^P(\mathbf{f}) - V^P(\mathbf{f} - f_j)$.

Furthermore, for all $\mathbf{f} \subseteq \mathbf{f}'$, $V^P(\mathbf{f}) \leq V^P(\mathbf{f}')$. (The more flow is added the higher the value for society due to increased number of rewards). Hence, the super game is a monotone utility game. Solving the super game via the modular subgames scheme with random ordering of the subgames corresponds to solving a monotone utility game where in each time step a player is chosen at random to play best response. We can apply Theorem 2. Due to the given homogeneous feasible partitioning, we can infer that the number of players in each subgame coordinated by a super player is ϑ. The total number of subgames is $\frac{n}{\vartheta}$, hence the solution is at least $\frac{\vartheta}{2n}$ times the social optimum. ∎

Note that instead of giving a bound for a homogeneous feasible subgames partitioning with respect to nodes in the game and number of nodes in the generated subgames, we could drop these specifications and just give a bound for a general feasible subgames partitioning with respect to the number of generated subgames. But we believe that the bound is more expressive with respect to the practical use in the given form.

IV. Experimental Evaluation

The paper closes with a comprehensive simulation study. In addition to the theoretical bounds proven in section III, we demonstrate here the quality of our novel decentralized modular subgames algorithm in experimental test. Simulations are performed using Matlab R2014b. The linear and quadratic programs within the algorithms are solved using Gurobi 6.0.4.

We evaluate the distributed modular subgames scheme on the balanced WSN routing game and the classical WSN routing game with subgames scheduling as presented in section III-C2 against the optimal centralized solution approach presented in section II-B, the social optimum of the classical WSN routing game, and two decentralized state-of-the-art algorithms. These are a decentralized scheme of the developers of the energy-balance optimal topologies from [3] which we denote as *Jarry11* and a state-of-the-art algorithm that exploits local information about node densities to achieve improved results presented in [5] which we denote as *Density12*. Our evaluation metrics are (1) energy balance, (2) network lifetime, and (3) energy efficiency which we define to be the total energy consumption of the network. Furthermore, we use experimental tests to draw conclusions on the relation of the energy-balance optimum and optimal solution of the classical WSN routing game and hence to the bound for the price of modularity from section III-C3.

A. Simulation Settings

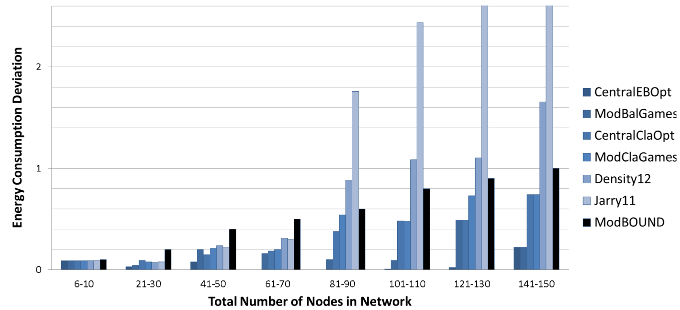
We conduct our experimental tests on WSNs instances described by an energy-balance optimal topology as introduced in section II. We choose a global node density (the fraction of number of nodes in the network and number of slices) of 15. With this, from the number of nodes n, we can generate the number of slices m equal to $\lceil n/15 \rceil$. The unique data gathering sink is placed in the center of the network with form of a circle, nodes are distributed randomly around the sink such that the indicated number of slices are each filled with at least one node and the network is fully connected. The initially available energy value of each node is 100. The data amount created by a node is a randomly assigned value from the interval $[1, 10]$. This describes a heterogeneous load distribution as usually the case in WSNs. The costs for sending one unit of flow on an edge between sensor nodes is 1. Costs for sending information on a direct edge to the sink can be inferred from the slice index and the assumed quadratic increase of costs with distance for radio transmission (i.e., costs for nodes in slice S_2 are 2^2, for nodes in S_3 are 3^2 and so on). In addition, we separate the network slices into 12 disjoint symmetric sectors (every 30 degrees of the circle a new sector begins). We will consider network instances with number of nodes between 6 and 150. For each interval, we randomly create 50 test instances. Per instance we conduct 60 iterations of the simulation to compute the average performance of an algorithm. We will depict results for the mean values for each metric in the following as results demonstrate strong concentration around the mean. For now, we are considering networks in the two dimensional plane, we anticipate that the transformation to three dimensions is straightforward. We will consider such instances explicitly in future work.

In our modular subgames algorithm on both games, we choose hop distance parameter $h = 2$ to make our algorithm comparable to the distributed benchmark algorithms. More precisely, we assume that communication is possible within 3 neighboring sectors. Subgames are formed of parallel sectors over $h + 1$ slices starting from the most inner slice.

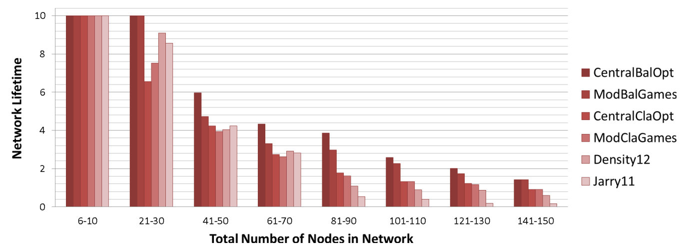
B. *Performances regarding Evaluation Metrics*

In Fig. 3 the results of our experimental tests for different intervals of total number of nodes in the network for the considered metrics are displayed. Please note the differing scales of the graphics in the figure.

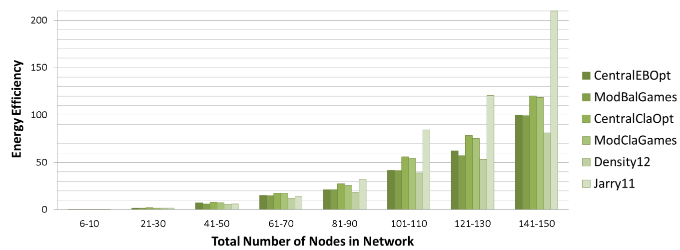
1) Energy Balance (Energy Consumption Deviation): Results for energy balance are given in Fig. 3a. First, we observe that energy balance in the centralized optimal solution (CentralEBOpt) varies over the different node intervals. In particular, the value becomes smaller from a situation with very few nodes in the network to at least 21 nodes. An explanation for this is the increased number of options to route flow, i.e., nodes that have already a high load factor may be avoided more easily. Solutions for the social optimum of the classical WSN routing game (CentralClaOpt) and also for its solution by the modular subgames scheme (ModClaGames) are initially poorer than those for the other distributed schemes. But upwards 60 nodes, both solutions become clearly better than the solutions of distributed benchmark schemes Jarry11 and Density12. The modular subgames scheme on the balanced WSN routing game (ModBalGames) outperforms all distributed schemes starting from 61 nodes and is always close to the optimal solution for energy balance. For more than 81 nodes, Jarry11 and Density12 are clearly above the absolute bound for the solutions of the modular subgames scheme on the balanced WSN routing game (ModBOUND) proven in section III-C2. Note that the weaker performance of Density12 compared to the results of the authors in [5] is due to the heterogeneous initial demands of the nodes. In



(a) Energy Balance



(b) Network Lifetime



(c) Energy Efficiency

Fig. 3. Experimental comparison of (a) energy balance, (b) network lifetime, and (c) energy efficiency (total energy consumption over all network nodes) of the solutions of the optimal centralized computation (CentralEBOpt), the modular subgames algorithm on the balanced WSN routing game (ModBalGames), the social optimum (CentralCalOpt) and the modular subgames algorithm on the classical WSN routing game (ModCalGames), and two distributed benchmark algorithms (Density12 & Jarry11) for networks of different sizes with heterogeneous node placement and message loads. In (a), the solutions are in addition compared against the absolute bound (ModBOUND) for ModBalGames.

their experimental evaluation all nodes had the same amount of initial messages. Furthermore, note that in the classical WSN routing game, every individual node uses less network information about past moves for routing than in the balanced WSN routing game (i.e., every node only needs data on current loads on edges of potential routes). The trade-off is a weaker performance that is still better than the one of the considered distributed benchmark schemes. The bound for the price of modularity of the classical WSN routing game guarantees that the solution via the modular subgames approach is close to the social optimum of this game. Hence, both have guaranteed comparable solutions also with respect to energy balance.[1]

[1]The bound for the price of modularity was explicitly provided for WSNs with a homogeneous node placement. In the experimental tests, we are considering heterogeneous node placements to extend knowledge on the performance of our scheme. When considering heterogeneous node placements, one can replace the parameters in the bound by the total number of generated subgames (see note at the end of section III-C3).

2) Network Lifetime: When evaluating the performances with respect to network lifetime, displayed in Fig. 3b, unlike as for energy balance, we measure individual node attributes. With increased number of nodes, the nodes usually have more options to evade heavily loaded nodes. Nevertheless, it cannot be avoided that nodes spend the more energy the more nodes, hence slices further away from the sink, compose the network. The overall picture is that with increased number of nodes network lifetime decreases in all solutions. Note that we are considering schemes for energy balance and thus only indirectly for network lifetime. While for 21 to 30 nodes the distributed benchmark schemes Density12 and Jarry11 outperform the solutions for the classical WSN routing game, between 41 and 70 nodes, all approaches, excluding the optimum for energy balance, perform relatively equal. Starting from 81 nodes the solutions for the classical WSN routing game are better than those for Density12 and Jarry11 and the modular subgames scheme (ModClaGames) is always close to the social optimum of this game (CentralCalOpt). The modular subgames scheme on the balanced WSN routing game (ModBalGames) outperformes all other schemes and is again close to the optimum for energy balance.

3) Energy Efficiency (Total Network Energy Consumption): In Fig. 3c the results for energy efficiency, defined as the sum of energy consumption over all nodes of the network, are presented. While Jarry11 is again clearly weaker than the other protocols, it is especially notable that the Density12 scheme performs in general even better than the optimal solution for energy balance (CentralEBOpt). The explanation for this is that minimizing the total energy consumption of the network is not equivalent to optimizing energy balance, respectively network lifetime. While the Density12 solution has lower total energy consumption, the network in general still dies faster than in the solution for optimal energy balance and than in the solutions for the modular subgames schemes due to overuse of some specific nodes. In the future, here it would certainly be interesting to vary the definition of lifetime and see how performances with respect to the different metrics change. In particular, it would be interesting to define critical nodes for network lifetime and to look into how far our modular subgames scheme can be adapted to it.

In summary, our simulation results supported our theoretical bounds for the distributed modular subgames scheme and showed that it also provides very good results in experimental tests. To be fair to the considered distributed benchmark algorithms, one must say that the modular subgames approach employs more information than the other schemes, i.e., updated information of current costs of potential routes. In practice, such information could be locally stored and updated by forwarding information peer to peer.

V. Conclusion

The overall goal in this paper was to create a good basis for the theoretical analysis of distributed energy balancing schemes for WSNs. We presented a novel approach for balancing energy in a distributed manner by solving a sequence of modular strategic subgames. We proved some theoretical bounds and demonstrated that the modular subgames scheme also performs good in experimental tests. Our scheme as well as the theoretical bounds are intended to be adapted gradually to more complex circumstances. Especially interesting are also the extension of our recent results to dynamic flows as well as to WSNs with mobile nodes. The principles of our model may also be useful for other balancing problems of decentralized resource usage.

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