

# INDOOR TRACKING WITH KALMAN FILTERS USING RSS-BASED RANGING

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# Chapter 1

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## Introduction

Indoor localisation and tracking have been growing in interest in the last few years due to their wide range of use cases such as indoor navigation, flow management in public places (e.g. train stations) and commercial uses (e.g. placements of ads, personalised advertisement). Due to the lack of a global solution such as Global Positioning System (GPS) for outdoor environments, indoor localisation is still considered as an unsolved problem. The increasing number of ubiquitous computing devices, especially for home automation, requires a low power consuming wireless standard. Currently most applications in this field are using the ZigBee standard for short range communications. The availability of low cost wireless devices distributed all over the house gives us the opportunity to use those sensors for localisation. Indoor localisation systems are divided into two classes: active terminal localisation and passive source localisation. In the active terminal localisation a device such as a mobile phone scans the packets from multiple signal sources (such as wireless access points or sensor nodes). It can then determine the Received Signal Strength (RSS) for each source. This information in addition to the known position of the signal sources can be used to determine the position of the device. In passive source localisation the targets are the source of the signals. A series of so called Anchor Nodes (ANs) are deployed in the environment. Those Anchor Nodes then passively overhear the signals of the signal source. Passive source localisation has the advantage that the node does not know that it is being localised. Therefore, it is more suitable to several business applications.

Indoor localisation is particularly challenging due to the complex propagation conditions of signals in indoor environments. The strong signal attenuation in indoor environments is normally caused by objects such as walls, bookcases, desks and the signal is heavily influenced by multipath propagation as well as always changing conditions (e.g. moving persons and objects).

In this thesis we use RSS-based passive localisation to determine the location of the target node. To do so we use Anchor Nodes with known locations to overhear the signals of the target node. RSS is measured by each AN and then converted into a distance information according to a suitable Path Loss model. This so called range information can then be used with the known position of each Anchor Node to apply a trilateration algorithm. Finally, we propose to apply Kalman Filters, including Kalman Filter and Extended Kalman Filter, to improve the obtained location information. The Kalman Filter uses a recursive procedure to process Gaussian noise in a discrete linear system, and Extended Kalman Filter can deal with non-linear systems. Kalman Filters provide an efficient computational method to estimate the state of the process.

In the remainder of this thesis, Chapter 2 provides the theoretical bases to this thesis. First we present two ranging models in Section 2.1, then we describe two versions of a trilateration algorithm called Linear Least Square (LLS) and Non-Linear Least Square (NLS) in Section 2.2 and finally we introduce the Kalman Filter (KF) and its non-linear form the Extended Kalman Filter (EKF) in Section 2.3. Chapter 3 will explain our used equipment in Section 3.1 followed by our data models. Chapter 4 will provide all information about our measurements and presents the evaluation of the measurements. Chapter 5 will conclude the thesis with a summary and some ideas for future works.

## Chapter 2

---

# Theoretical Foundations of Indoor Localisation and Kalman Filters

In this chapter, all the necessary backgrounds for this thesis will be presented and explained. It is divided into three parts. First, the Path Loss models are explained in Section 2.1. Second, the used localisation algorithms are presented in Section 2.2. Finally, in Section 2.3 we present the Kalman Filter and its non-linear form the Extended Kalman Filter.

### 2.1 Signal Propagation

Path Loss describes the attenuation of a wireless signal over the propagation distance. This attenuation is the result of multiple influences to the signal such as objects, buildings and the propagation distance. In an open space without any obstacles the most important factor to the attenuation is the propagation distance. We can derive a direct relation between the strength of the signal and the propagation distance. Usually we use the Log-normal Distance Path Loss model to model this relation.

In indoor environments there are more factors than the propagation distance that influence the attenuation of the signal strength. It gets influenced by walls, furniture, people and animals. We distinguish between Line Of Sight (LOS) and Non Line Of Sight (NLOS) signal propagation. If there is a Line Of Sight between the target and an Anchor Node the Received Signal Strength is normally stronger compared to NLOS, as the signal does not have to pass several obstacles (e.g. walls, furniture). Furthermore, the signal gets reflected from walls, and furniture. This effect is called Multi-Path Propagation. The signal of a node arrives at an Anchor Node (AN) multiple times with different strength. For example it reaches the node directly over the air and once on an indirect path reflected of a wall. The multiple signals then blend together to one, resulting in a large amount of fluctuation in the measured power of the signal at AN. We refer to this measured signal strength as Received Signal Strength (RSS). Due to these influences it is more challenging to find the relation between RSS and the propagation distance in indoor environments than in outdoor environments where the propagation distance is the main attenuation factor.

### 2.1.1 Log-normal Distance Path Loss

The most commonly used model of the relation between the propagation distance  $d$  and RSS is called Log-normal Distance Path Loss (LDPL) and is defined by [1]:

$$PL = PL_0 + 10\gamma \log_{10} \left( \frac{d}{d_0} \right) + X_g. \quad (2.1)$$

PL is the path loss measured in Decibel (dB),  $\gamma$  is the path loss exponent,  $PL_0$  is the path loss at reference distance  $d_0$  and  $X_g$  references to some Gaussian noise with zero mean. It is a very generic model and is designed to fit a wide range of environments.

Equation 2.1 can be simplified using one meter as reference distance  $d_0$  and defining constants  $\alpha$  and  $\beta$ .

$$\alpha = PL_0 \quad (2.2)$$

$$\beta = 10\gamma \quad (2.3)$$

Without considering the Gaussian noise, Equation 2.1 can be rewritten as:

$$PL = \alpha + \beta \log_{10}(d) \quad (2.4)$$

To derive the LDPL constants  $\alpha$  and  $\beta$  typically linear regression is used [2], [3].

### 2.1.2 Non-linear Regression

LDPL is a very generic model for Path Loss prediction and is applicable on a wide range of environments. Therefore, LDPL is widely used to predict the Path Loss. It has been shown to be inaccurate for indoor environments. In [4] the authors proposed a Path Loss model based on Non-linear Regression (NLR). The relationship between RSS and distance is modelled as following:

$$d_i = \alpha_i e^{\beta_i RSS_i} \quad (2.5)$$

where  $d_i$  is the distance between the  $i$ -th AN and the signal source,  $RSS_i$  is the RSS at  $i$ -th AN and  $\alpha_i$ ,  $\beta_i$  are AN specific coefficients.  $\alpha$  and  $\beta$  can be obtained by preliminary measurements in the target area. To obtain the constants  $\alpha$  and  $\beta$  a non-linear least square function fitting is performed. The non-linear least square curve fitting is mainly achieved by either using a trust region or performing a linear search. When using a trust region the error is assumed to be Gaussian and 99% of errors are smaller than 2.5 times the estimated standard error. We accept the result when it is in our trust region and therefore, in a range around our estimated value. When solving the curve fitting with a linear search, the possible solutions are computed iteratively and then searched for the best fitting result. To ensure the algorithm finishes some boundaries can be set.

## 2.2 Localisation Algorithms

To determine the location of the target we used the following two kinds of trilateration algorithms.

### 2.2.1 Linear Least Square

In least square based localisation we determine the position of a node based on its distance to various Anchor Nodes (ANs). This distance vector  $d_i$  is derived from a Path Loss model as described in Section 2.1. With this information we can obtain the maximum likelihood solution by using a Non-Linear Least Square approach. To estimate the position of the node  $(\hat{x}, \hat{y})$  we need to solve the following equation system, assuming 5 ANs are deployed.

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} \sqrt{(\hat{x} - AN_{1x})^2 + (\hat{y} - AN_{1y})^2} \\ \sqrt{(\hat{x} - AN_{2x})^2 + (\hat{y} - AN_{2y})^2} \\ \sqrt{(\hat{x} - AN_{3x})^2 + (\hat{y} - AN_{3y})^2} \\ \sqrt{(\hat{x} - AN_{4x})^2 + (\hat{y} - AN_{4y})^2} \\ \sqrt{(\hat{x} - AN_{5x})^2 + (\hat{y} - AN_{5y})^2} \end{pmatrix} \quad (2.6)$$

As the distance vector  $d$  is noisy we want to obtain an optimal solution by minimising the squared error of the system. This gives us the following problem to solve:

$$(\hat{x}, \hat{y}) = \operatorname{argmin}_{x,y} \sum_{i=1}^N \left[ \sqrt{(AN_{ix} - x)^2 + (AN_{iy} - y)^2} - d_i \right]^2 \quad (2.7)$$

where  $N$  is the amount of ANs and  $AN_i$  is the coordinate of the  $i$ -th AN.

Solving the equation system can either be done with complex computational methods such as the steepest descent or the Gauss-Newton techniques. To avoid this computational complexity we use a Linear Least Square approach as proposed in [5]. In this approach the expression of one Anchor Node is subtracted from all other expressions to obtain linear relations. Various versions have been studied in [6] and [7]. As demonstrated in [8] we subtract a random measurement  $r$  from all other equations. We can transform Equation 2.6 to a matrix formulation:

$$Al = p, \quad (2.8)$$

where  $l = [x, y]^T$ , with:

$$\mathbf{A} = 2 \cdot \begin{bmatrix} AN_{1x} - x_r, AN_{1y} - y_r \\ \vdots \\ AN_{r-1x} - x_r, AN_{r-1y} - y_r \\ AN_{r+1x} - x_r, AN_{r+1y} - y_r \\ \vdots \\ AN_{nx} - x_r, AN_{ny} - y_r \end{bmatrix}, \quad (2.9)$$

$$\mathbf{p} = \begin{bmatrix} z_r^2 - z_1^2 - k_r + k_1 \\ \vdots \\ z_r^2 - z_{r-1}^2 - k_r + k_{r-1} \\ z_r^2 - z_{r+1}^2 - k_r + k_{r+1} \\ \vdots \\ z_r^2 - z_n^2 - k_r + k_n \end{bmatrix}, \quad (2.10)$$

$k_i = x_i^2 + y_i^2$  and  $r$  is the reference equation.

Given the formula in Equation 2.8, we can solve it for  $l$  as follows:

$$\hat{l} = (A^T A)^{-1} A^T p = [x, y]^T. \quad (2.11)$$

As LLS solves the equation system in a linear way we expect it to introduce some errors to our localisation. However, we can benefit from very low computational complexity.

## 2.2.2 Non-Linear Least Square

Non-Linear Least Square (NLS) is a generalised version of the Linear Least Square problem. For localisation it solves Equation 2.6 but in a non-linear way. Most implementations of NLS are recursive and compute the result numerically. This is very compute intense and, if the initial guess is bad, may take quite long. Despite this disadvantage is NLS more accurate than LLS as it is able to find solutions with smaller errors.

## 2.3 Kalman Filters

In 1960, Rudolf E. Kalman published a paper describing a recursive algorithm to solve the discrete-data linear filtering problem [9]. Since then the so called Kalman Filter (KF) has been subject of extensive research and a wide variety of applications.

The Kalman Filter is a set of equations that predicts the next state of a system in a way that minimises the mean of the squared error. KF consists of a two-step procedure: the state estimation and the state update which is based on information of a measurement.

### 2.3.1 Linear Kalman Filter

The Kalman Filter (KF) is a well known equation system and is widely used in many applications. Kalman Filter has been a subject of extensive research in the past years [10], [11], [12], [13], [14].

For Kalman filtering we need a system consisting of two things: a state vector  $x$  and a measurement vector  $z$ , where  $x_k$  (the state  $x$  at time  $k$ ) and  $z_k$  (the measurement  $z$  at time  $k$ ) are defined as follows:

$$x_k = Ax_{k-1} + w_{k-1} \quad (2.12)$$

$$z_k = Hx_k + v_k \quad (2.13)$$

$w_{k-1}$  and  $v_k$  represent the process and measurement noise. They are assumed to be independent and Gaussian with zero mean.

$$p(w) \sim N(0, Q), \quad (2.14)$$

$$p(v) \sim N(0, R). \quad (2.15)$$

The Kalman Filter is based on a two-step procedure. The first step is called state prediction and consists of the following two equations:

$$\hat{x}_k = A\hat{x}_{k-1} \quad (2.16)$$

$$P_k = AP_{k-1}A^T + Q \quad (2.17)$$

In this step Equation 2.16 estimates the new state of the system and Equation 2.17 projects the covariance estimate forward.

The second step consists of the following three equations:

$$K_k = P_k H^T (H P_k H^T + R)^{-1} \quad (2.18)$$

$$\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k) \quad (2.19)$$

$$P_k = (I - K_k H) P_k \quad (2.20)$$

In this step Equation 2.18 calculates the Kalman gain and Equation 2.19 actually takes the current measurement  $z_k$  and incorporates it with the predicted state. Equation 2.20 updates the state error covariance.

The Kalman Gain  $K$  is the most important part of KF as it controls the discrepancy between the actual measurement  $z_k$  and the estimated state of the system  $H\hat{x}_k$ .  $K$  minimises the error covariance  $P$  (Equation 2.18).

Kalman Filter is the optimal solution to estimate the state of a linear system with Gaussian noise. In Section 3.3 we will present a system that models the position and movement of our node. We will use this system to filter the positions calculated with the Least Squares algorithms. We expect the filtered positions to be more accurate than the original positions.

### 2.3.2 Extended Kalman Filter

Although Kalman Filter is very powerful it is limited by the linearity of the state and the measurements (Equations 2.12 and 2.13). Therefore, a non-linear extension of Kalman Filter has been developed. In Extended Kalman Filter (EKF) the system is not assumed to have a linear relation between the previous state and the current state. This results in the following change to Equation 2.12:

$$x_k = f(x_{k-1}, w_{k-1}) \quad (2.21)$$

and the measurement (Equation 2.13) is changed to:

$$z_k = h(x_k, v_k) \quad (2.22)$$

where  $f$  and  $h$  are non-linear functions. The function  $f$  relates the previous state  $x_{k-1}$  to the current state  $x_k$  and the Gaussian zero-mean process noise  $w_{k-1}$ . The function  $h$  relates the state  $x_k$  to the measurement  $z_k$ . As we do not know the noise in practice we can approximate the state and measurement vector without it:

$$\tilde{x}_k = f(x_{k-1}, 0), \quad (2.23)$$

$$\tilde{z}_k = h(x_k, 0). \quad (2.24)$$

To estimate the process with these non-linear relations we need to change our equation system by some local linearisation methods. Equation 2.23 and 2.24 can be linearised as:

$$x_k \approx \tilde{x}_k + A_k(x_{k-1} - \hat{x}_{k-1}) + W_k w_{k-1}, \quad (2.25)$$

$$z_k \approx \tilde{z}_k + H_k(x_k - \tilde{x}_k) + V_k v_k, \quad (2.26)$$

where  $x_k$  is the actual state,  $z_k$  is the actual measurement,  $\tilde{x}_k, \tilde{z}_k$  are their approximations,  $\hat{x}_{k-1}$  is the final state estimate of the previous computation,  $w_k$ , and  $v_k$  represent the noise as in Equation 2.14 and 2.15.  $A$  is the Jacobian matrix of partial derivatives of  $f$  with respect to  $x$ ,  $W$  is the Jacobian matrix of partial derivatives of  $f$  with respect to  $w$ ,  $H$  is the Jacobian matrix of partial derivatives of  $h$  with respect to  $x$  and  $V$  is the Jacobian matrix of partial derivatives of  $h$  with respect to  $v$ . We define the prediction error as

$$\tilde{e}_{x_k} \equiv x_k - \tilde{x}_k, \quad (2.27)$$

and the measurement residual as

$$\tilde{e}_{z_k} \equiv z_k - \tilde{z}_k. \quad (2.28)$$

We can write the equations for the process error as

$$\tilde{e}_{x_k} \approx A(x_{k-1} - \hat{x}_{k-1}) + \varepsilon_k, \quad (2.29)$$

$$\tilde{\epsilon}_{z_k} \approx H\tilde{\epsilon}_{x_k} + \eta_k, \quad (2.30)$$

where  $\epsilon_k$  and  $\eta_k$  represent new independent random variables with zero mean. We can use the measurement residual  $\tilde{\epsilon}_{z_k}$  to estimate the prediction error  $\tilde{\epsilon}_{x_k}$ . We call this estimate  $\hat{\epsilon}_k$ . The estimation could be performed in a second (hypothetical) Kalman Filter.

$$\hat{x}_k = \tilde{x}_k + \hat{\epsilon}_k \quad (2.31)$$

Assuming the predicted value of  $\hat{\epsilon}_k$  is zero, the equation for the estimate would be:

$$\hat{\epsilon}_k = K_k \tilde{\epsilon}_{z_k}. \quad (2.32)$$

By substituting Equation 2.32 back into Equation 2.31 we see that we do not need the second (hypothetical) Kalman Filter:

$$\hat{x}_k = \tilde{x}_k + K_k \tilde{\epsilon}_{z_k}. \quad (2.33)$$

We then obtained the complete set of equations for the Extended Kalman Filter. The state prediction equations:

$$\hat{x}_k = f(\hat{x}_{k-1}), \quad (2.34)$$

$$P_k = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T, \quad (2.35)$$

and the state update equations:

$$K_k = P_k H_k^T (H_k P_k H_k^T + V_k R_k V_k^T)^{-1}, \quad (2.36)$$

$$\hat{x}_k = \hat{x}_k + K_k (z_k - h(\hat{x}_k)), \quad (2.37)$$

$$P_k = (I - K_k H_k) P_k. \quad (2.38)$$

In Chapter 3 we will introduce a possible system to use EKF for localisation. In our proposed system we do not need an explicit localisation algorithm as EKF is able to use the non-linear relation between position and distances to the various ANs.



## Chapter 3

---

# Tracking System Implementation

In this Chapter we will introduce our used equipment and software (Section 3.1). Then we will present our models used to perform the localisation process in Section 3.2. Finally we will present the state model used for Kalman Filter in Section 3.3 and the non-linear model used with Extended Kalman Filter in Section 3.4.

### 3.1 Used Equipment and Software

We deployed the system [15] on the third floor in the building of the IAM institute at University of Bern. The system consists of five ANs and one sensor node. As ANs we used Universal Software Radio Peripheral (USRP) N210 devices and as the target node we used a TelosB sensor. The TelosB sensor was configured to send packets at a constant rate of 5 packets per second.

The RSS data is collected with a passive system based on Software Defined Radio. The TelosB sensor sent packages over IEEE 802.15.4 which are overheard by ANs. The signals from ANs is then decoded by a computer with GNU Radio and then filtered for RSS. This RSS information is then processed by a script in MATLAB.

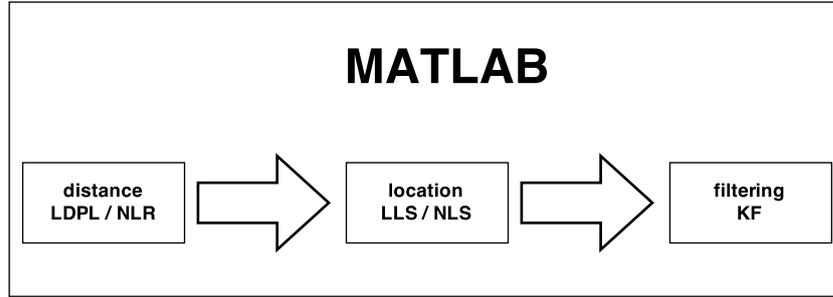
**Figure 3.1:** Data Flow



### 3.2 Calculation Model

To analyse the measured data in MATLAB we use a three step procedure as shown in Figure 3.2. First we determine the distance of the target node to each AN. Then we apply a localisation algorithm, i.e. LLS and NLS, to determine the location of the target node. Finally we apply a filter to this location information to improve our initial localisation.

**Figure 3.2:** Localisation Steps in MATLAB with KF



The first step in our analysis is to find the distances of the target node to at least three ANs. With this information we can obtain the location of the target node by applying a range based localisation algorithm. To get this distance information we used one of the Path Loss (PL) models described in Section 2.1. Equation 2.5 (NLR) is already solved for distance but for the LDPL model we have to solve Equation 2.4 for distance. This simple task gives us the following equation:

$$d = 10^{\frac{RSS - \alpha}{\beta}} \quad (3.1)$$

Now that we have the distances between the node and ANs we can now apply a range-based localisation algorithm.

### Localisation

To transform the distance information to a location we need to know the positions of our ANs. Those coordinates are measured using a laser distance meter. The positions of the ANs are as following:

**Table 3.1:** The Coordinates of each AN

|              | AN 1 | AN 2 | AN 3 | AN 4 | AN 5  |
|--------------|------|------|------|------|-------|
| x-coordinate | 14.4 | 14.6 | 10.7 | 1.16 | 4.75  |
| y-coordinate | 1.04 | 14.9 | 3.46 | 1.07 | 15.85 |

With the distance to each AN and the known positions of each AN we can now use the algorithms, i.e. LLS and NLS, described in Section 2.2 to derive the location of the target node. The localisation error describes the difference between the ground truth position  $(x, y)$  of the target node and the calculated position  $(\hat{x}, \hat{y})$ .

$$E = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2} \quad (3.2)$$

We will later use this parameter to compare the performance of the different methods of localisation.

### 3.3 State Model for Kalman Filter

To further improve our initial estimated position by trilateration algorithms we can use several filter algorithms. In this thesis we focus on Kalman Filter. To do so we first need to define our system as described in Section 2.3. The state  $x$  of our system is defined as

$$x = \begin{pmatrix} x_{position} \\ x_{velocity} \\ y_{position} \\ y_{velocity} \end{pmatrix}, \quad (3.3)$$

our measurement  $z$  is defined as:

$$z = \begin{pmatrix} x_{position} \\ y_{position} \end{pmatrix}. \quad (3.4)$$

The matrices  $A$  and  $H$  are defined as

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3.5)$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (3.6)$$

Finally we need to set the appropriate measurement and process covariances. If we would know the ground truth position we could compute the localisation error for our measurement and use those to calculate the appropriate covariances. Since our objective is to find the current position of the target node we can only assume those values. We can estimate the state covariance based on the information that we track a walking person and therefore, the speed should be between 0.5 and 1.5 meters per second. As we trust our localisation measurements less we assume the error to be big. We set the covariance matrices as following:

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (3.7)$$

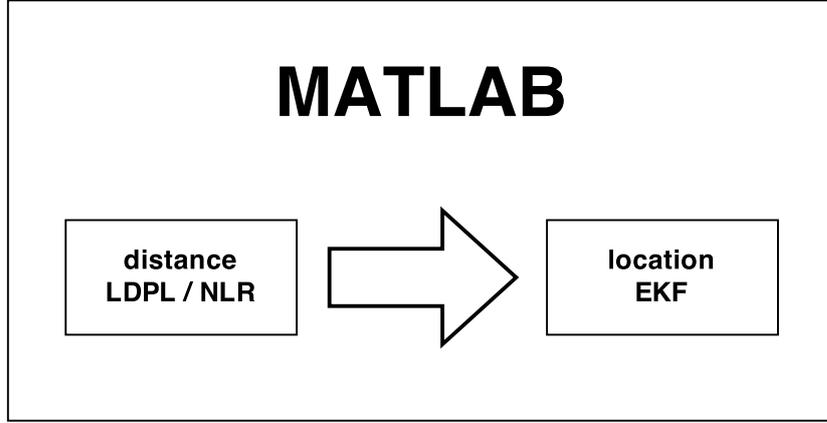
$$R = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1.5 \end{pmatrix}. \quad (3.8)$$

To make the results for the various algorithms comparable we keep those matrices constant.

### 3.4 State Model for Extended Kalman Filter

Due to the linearity of Equations 2.12 and 2.13 Kalman Filter has a low accuracy for non-linear state and measurement conditions. Therefore, we evaluated also the Extended Kalman Filter (EKF). As described in Section 2.3.2 the main differences are the state and measurement equations, which now contain functions instead of matrices. This means we can directly derive the location information within EKF without the need for a separate localisation algorithm. This reduces our errors as we lose one component in our localisation procedure (Figure 3.2). The procedure for EKF is shown in Figure 3.3.

**Figure 3.3:** Localisation Steps in MATLAB with EKF



To simplify the comparison between the two versions of KF we keep the state as defined in Equation 3.3. Therefore, the function  $f$  is defined as

$$x_k = Ax_{k-1}. \quad (3.9)$$

Instead of relying on a localisation algorithm we can directly use the distance information provided by ANs. This means we define the function  $h$  as following:

$$z_k = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = h(x_k) = \begin{pmatrix} \sqrt{(x_x - AN_{1x})^2 + (x_y - AN_{1y})^2} \\ \sqrt{(x_x - AN_{2x})^2 + (x_y - AN_{2y})^2} \\ \sqrt{(x_x - AN_{3x})^2 + (x_y - AN_{3y})^2} \\ \sqrt{(x_x - AN_{4x})^2 + (x_y - AN_{4y})^2} \\ \sqrt{(x_x - AN_{5x})^2 + (x_y - AN_{5y})^2} \end{pmatrix} \quad (3.10)$$

This change requires us to adapt the measurement covariance to base on the range error of our PL model instead of the localisation error. We predict the range error to be big and set the matrix to

$$R = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}. \quad (3.11)$$

As before we set these values to constants as we do not have enough information to compute the real values.



## Chapter 4

# Measurement Setup and Evaluation

In this chapter we evaluate the performance of Kalman Filter and Extended Kalman Filter by performing measurements in our institute. First we will present our measurement setup and the generated data set. Second we will compare the performance of the two presented Path Loss models based on their range errors. Third we will compare the performance of the localisation algorithms before and after applying Kalman Filter. Finally we will compare the performance of Kalman Filter and Extended Kalman Filter.

### 4.1 Setup

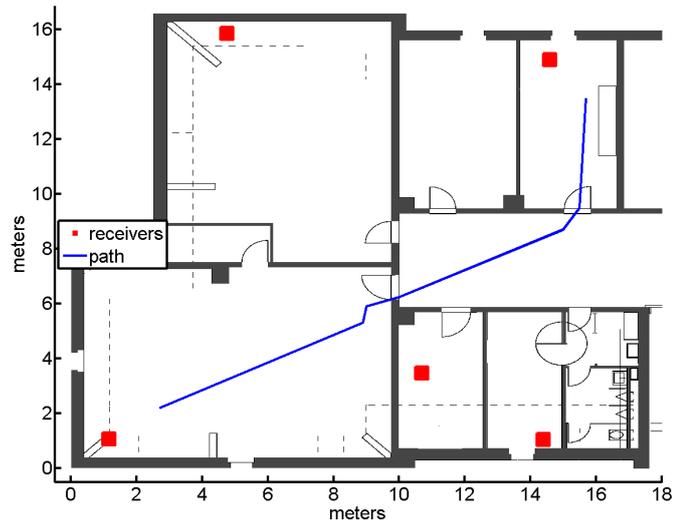
**Figure 4.1:** Deployed ANs in the Measurement Area



The Anchor Nodes (ANs) were deployed on the third floor of IAM building at University of Bern. Figure 4.1 shows the floor plan and the deployed ANs.

To ensure a consistent path in the measurement some points with known positions are marked along the moving path as the ground truth positions. The path used for this measurement can be found in Figure 4.2.

**Figure 4.2:** The Path Followed During the Measurement



## 4.2 Measurement Procedure

The measurements were started at the bottom left point of the path shown in Figure 4.2. The TelosB sensor node was powered up and then reset using its built in reset button. After resetting, the person who holds the TelosB sensor starts to move along the moving path. This procedure was repeated ten times with speeds varying between 0.5 to 1.5 meters per second. During the measurement the five USRP N210 ANs recorded all the wireless traffic of the target node. This data was sent to a workstation over LAN. At the workstation the data was filtered for the relevant information (RSS) which was stored in text files for further analysis. The text files were then imported into MATLAB to aggregate the information and apply the localisation steps consisting of ranging using the two different Path Loss models and localisation (LLS, NLS) as well as filtering steps. To minimise the general fluctuation in the signal strength we averaged the RSS values of one second (five packets).

## 4.3 Path Loss Determination

To derive the Path Loss models in this specific setup we used the results of our last measurement run with the moving speed of 0.8 meters per second along the moving path in Figure 4.2. The values are determined with polynomial curve fitting for LDPL and with non-linear regression

curve fitting for NLR. The values were determined per AN and are as stated in Tables 4.1 and 4.2.

**Table 4.1:** LDPL Constants

|          | AN 1    | AN 2    | AN 3    | AN 4    | AN 5    |
|----------|---------|---------|---------|---------|---------|
| $\alpha$ | -12.697 | -15.408 | -23.489 | -25.168 | 5.557   |
| $\beta$  | -36.623 | -31.547 | -26.154 | -22.886 | -49.426 |

**Table 4.2:** NLR Constants

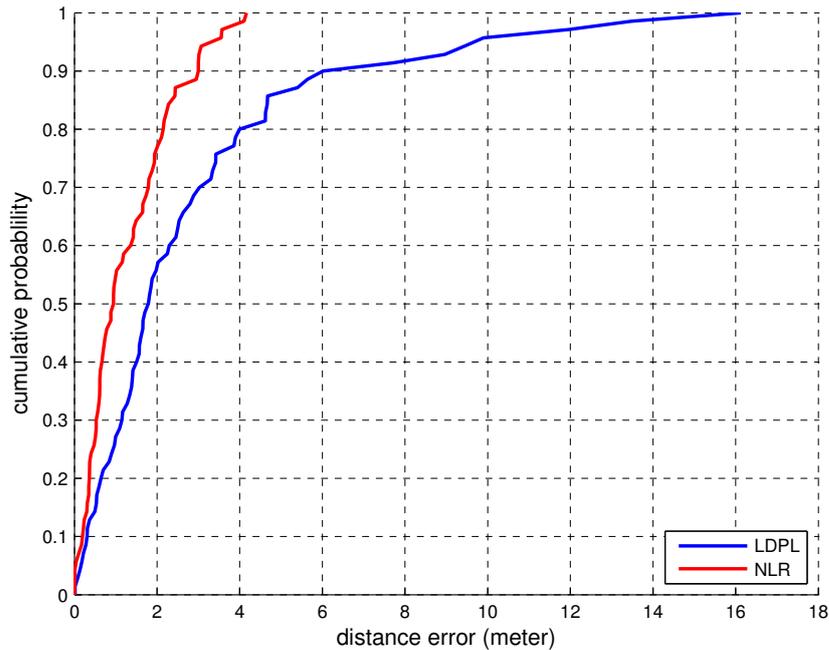
|          | AN 1     | AN 2     | AN 3     | AN 4     | AN 5      |
|----------|----------|----------|----------|----------|-----------|
| $\alpha$ | 2.416    | 0.4632   | 0.5942   | 0.5972   | 10.08     |
| $\beta$  | -0.02729 | -0.06538 | -0.05281 | -0.05995 | -0.003419 |

Using multiple measurement runs along different moving paths to determine the PL model would result in a flatter curve because the distance attenuation in a small area is very low and because of the signals heavy fluctuations caused by the indoor environment. This would result in higher ranging errors and therefore, reduce the overall performance of the localisation significantly. By using the same path to determine PL as we use for the evaluation of the localisation performance we receive very specific PL values which are only suitable for our measurement area. Using one measurement with the same moving pattern as the measurements we will analyse will give us the advantage to be more precise than a more generic determination of the coefficients. Since in this work, we are planning to analyse the performance of different localisation and tracking filters, using the single moving path to obtain PL will not influence the evaluation and can help us isolate the influence of the inaccurate ranging. For our further analysis of the localisation performance we used different measurement runs.

## 4.4 Performance of the Different Path Loss Models

As discussed in Section 2.1 it is very difficult to model the signal propagation of indoor environments as the varying amount of walls and obstacles and the multipath propagation influence the signal very heavily.

**Figure 4.3:** CDF of the Path Loss Models



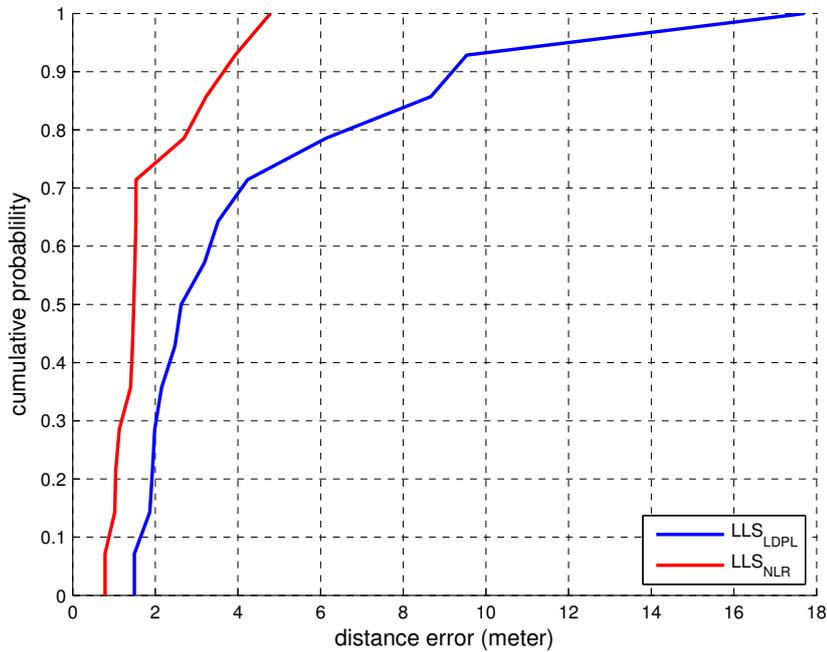
One way to visualise the performance of the Path Loss models is to plot the Cumulative Distribution Function (CDF) of the range errors. Figure 4.3 presents the probability of the distance error in meters. About 50% of all distance errors are less than 0.95 meters for NLR and less than 1.8 meters for LDPL. At 80% the difference between the two models gets even bigger. The distance error for NLR is 2.1 meters and for Log-normal Distance Path Loss it is 4 meters. Clearly the NLR model is more accurate than LDPL as the maximal errors are 4.1 meters for NLR and 16.1 meters for LDPL.

To give a better idea how this difference in range error influences the localisation step, we compare the localisation results for LLS and NLS. Figures 4.4 and 4.5 show the corresponding CDF for the localisation.

### Linear Least Square

Clearly the NLR model provides better localisation results than the LDPL model. For LLS 50% of localisation errors with NLR are smaller than 1.7 meters and for LDPL smaller than 3.6 meters. At 80% the difference is even bigger: 2.5 meters versus 6.7 meters for NLR and LDPL

**Figure 4.4:** CDF of the Linear Least Square Computed Locations



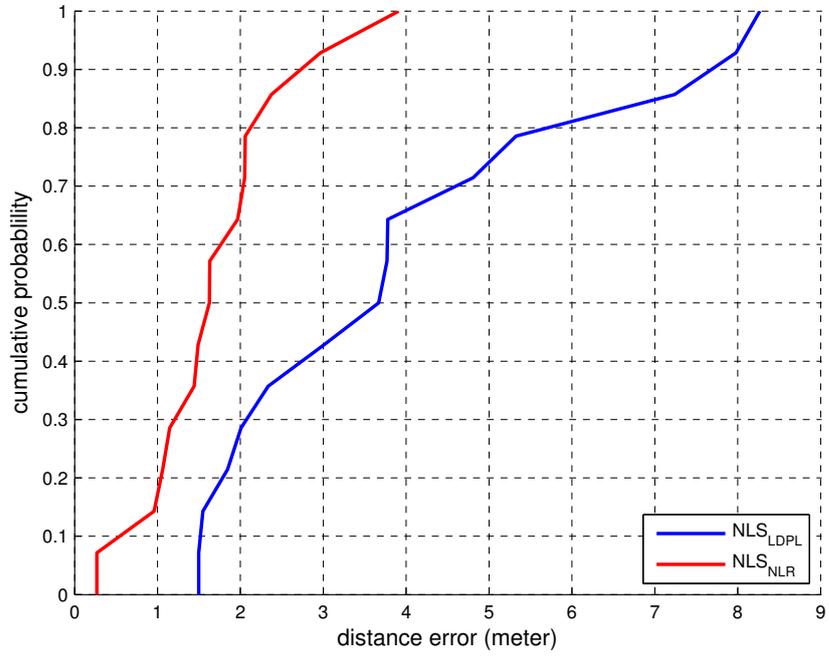
respectively. While NLR has a maximum error of 3.6 meters LDPL has errors as high as 17.7 meters. This error is greater than our measurement area.

### Non-Linear Least Square

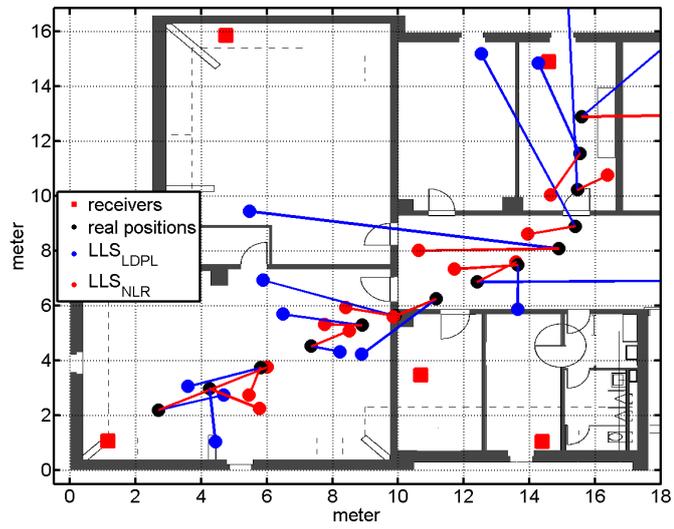
For NLS the results are much better as NLS computes the localisation using non-linear methods to minimise the squared errors. It reaches at 50% errors up to 1.6 meters for NLR and up to 3.6 meters for LDPL. At 80% the errors are up to 2.1 meters for NLR and 5.6 meters for LDPL. Although we will compare LLS and NLS later in this chapter it is worth to point out that the maximal localisation error for NLS (about 8.3 meters) is much smaller than the localisation error of LLS (about 17.8 meters) when using Log-normal Distance Path Loss. This difference is the result of the linearity respectively non-linearity of finding the optimal solution.

For further comparison we have plotted the localisations on our floor plan. Figure 4.6 and 4.7 show the calculated positions and the ground truth positions for the two Path Loss models.

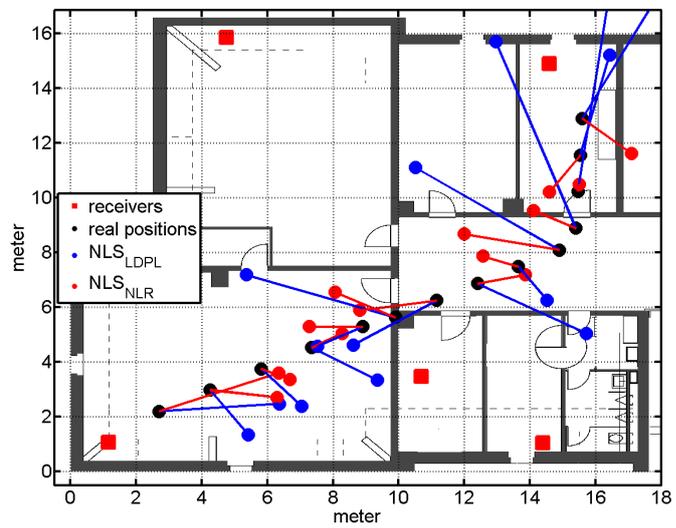
**Figure 4.5:** CDF of the Non-Linear Least Square Computed Locations



**Figure 4.6:** Calculated Locations based on LLS using both PL Models



**Figure 4.7:** Calculated Locations based on NLS using both PL Models



## 4.5 Performance of Kalman Filter

To evaluate if the Kalman Filter can improve the localisation we compare the localisation errors with and without using KF.

### 4.5.1 Linear Least Square

First we compare the results for the locations computed using LLS.

Log-normal Distance Path Loss

**Figure 4.8:** CDF of the Linear Least Square with and without KF for LDPL

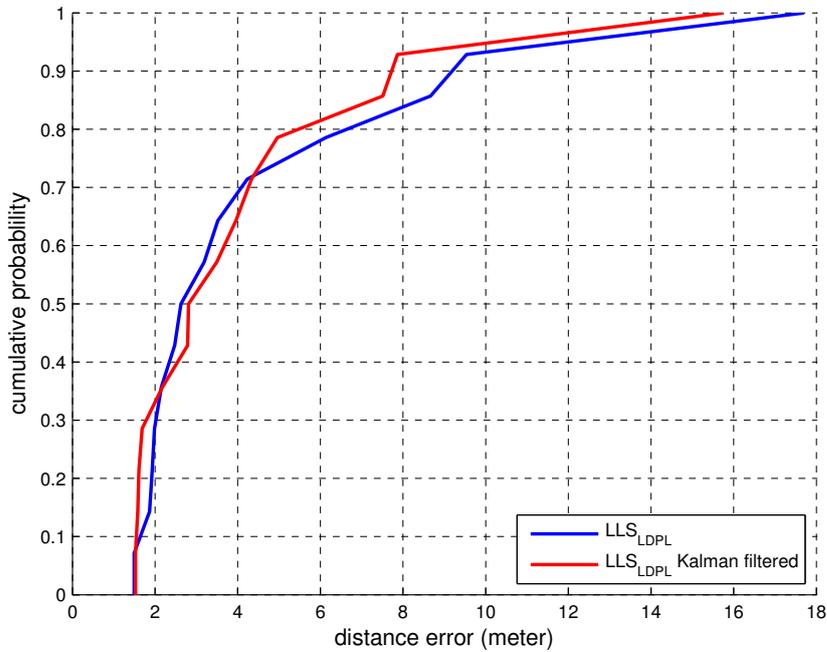
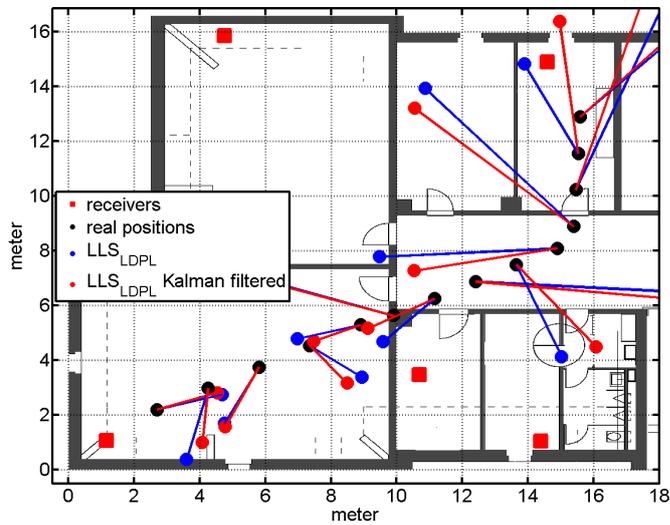


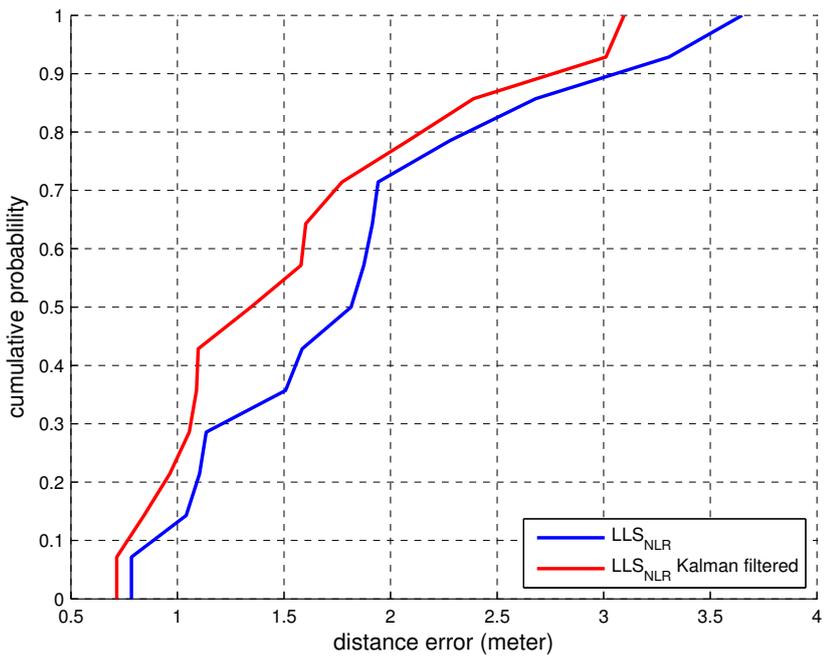
Figure 4.8 shows CDF of the location errors from LLS using LDPL. 50% of the errors are less than 3.6 meters for the filtered positions and only 0.1 meters worse when not filtered. 80% of the filtered positions have errors up to 5.6 meters. 80% of the unfiltered positions have errors up to 6.7 meters. The maximum error of LLS with KF is with 15.7 meters lower than the unfiltered positions which have errors up to 17.7 meters. Although the curves seem very similar, the filtered positions are more accurate than the unfiltered positions.

Figure 4.9 shows the ground truth positions and the filtered as well as the unfiltered LLS locations.

**Figure 4.9:** Map of the Linear Least Square Computed Locations for LDPL



**Figure 4.10:** CDF of the Linear Least Square with and without KF for NLR

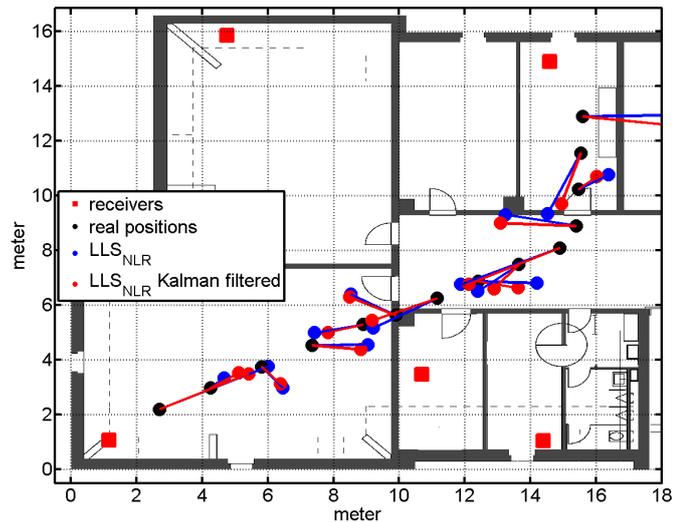


### Non-linear Regression

The red curve in Figure 4.10 for KF is clearly better than the curve for LLS (in blue). 50% of errors for KF are less than 1.3 meters meanwhile LLS has errors up to 1.8 meters at 50%. Both

curves are close to each other at 80% with only a difference of about 0.2 meters. The maximum error for KF is clearly better with up to 3.1 meters compared to the 3.6 meters maximum error of LLS.

**Figure 4.11:** Map of the Linear Least Square Computed Locations for NLR



## 4.5.2 Non-Linear Least Square

### Log-normal Distance Path Loss

In Figure 4.12 CDF of the NLS localisation errors are shown. 50% of the errors for KF filtered positions are smaller than 3 meters meanwhile the unfiltered positions have errors up to 3.7 meters. At 80% the errors are less than 5.1 meters for the filtered positions and less than 5.1 meters for the unfiltered positions. Both have a maximum error of 8.2 meters.

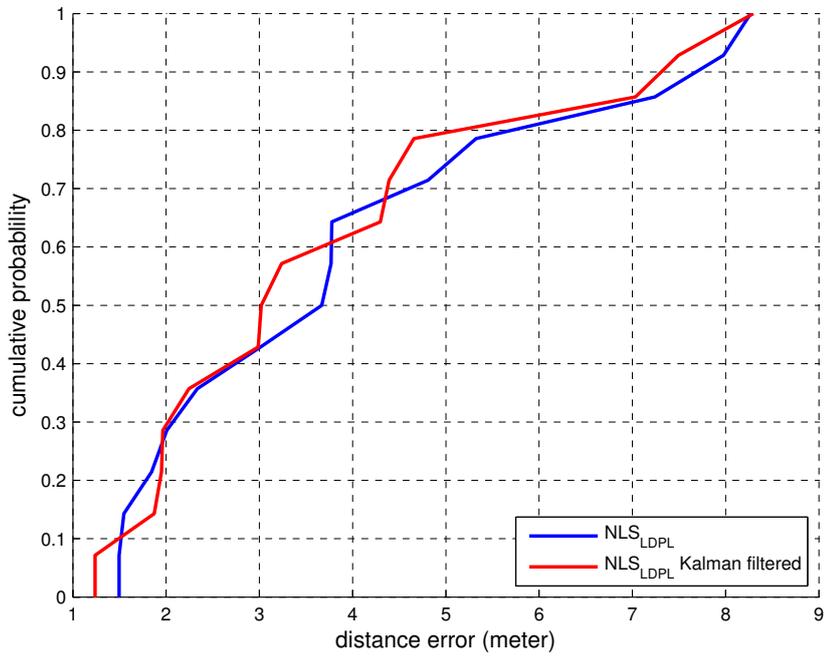
Figure 4.13 shows the ground truth positions and the filtered as well as the unfiltered NLS locations.

### Non-linear Regression

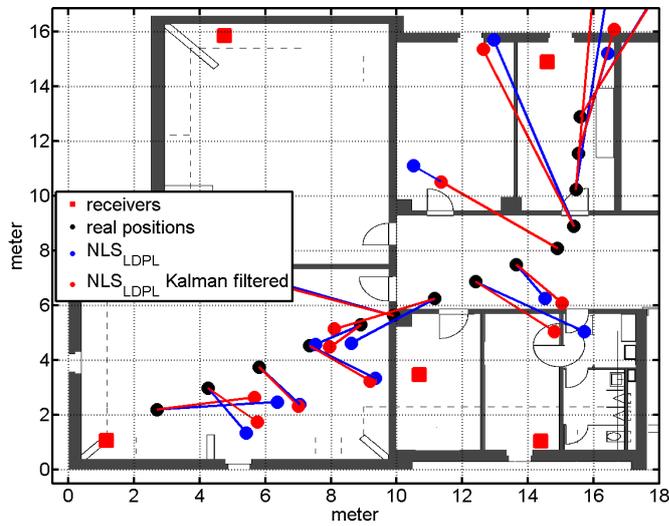
In Figure 4.14 CDFs of the localisation errors of NLS with and without KF are shown. Again the results for the filtered locations are better than without the filter. At 50% the errors are below 1.4 meters for KF and below 1.6 meters for NLS.

As expected Kalman Filter is able to decrease the localisation errors for Least Square based localisation. It can do so because Kalman Filter estimates the next position based on the previous position and its knowledge about the errors (state and measurement covariances). Kalman Filter is generally conservative to change and stays more or less (depending on the calculated Kalman

**Figure 4.12:** CDF of the Non-Linear Least Square with and without KF for LDPL

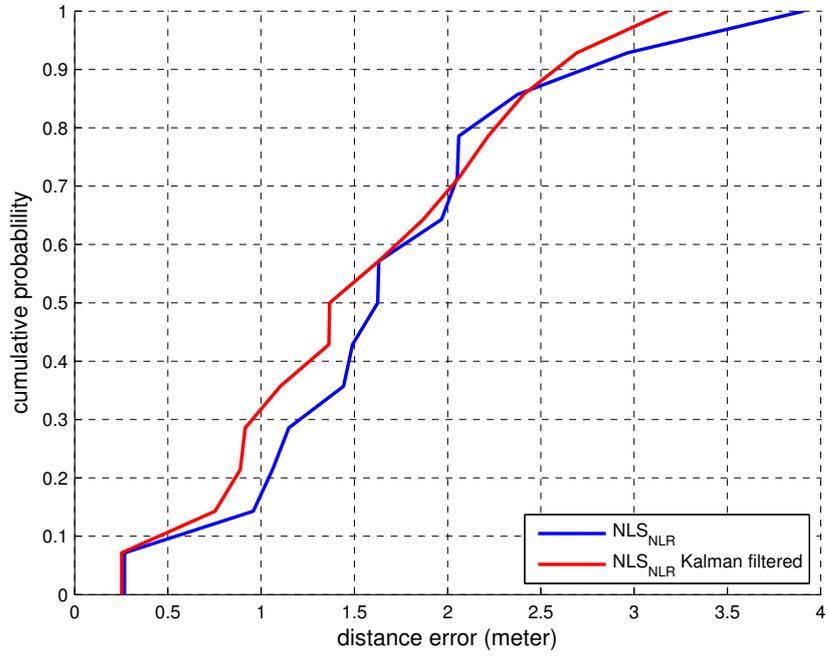


**Figure 4.13:** Map of the Non-Linear Least Square Computed Locations for LDPL

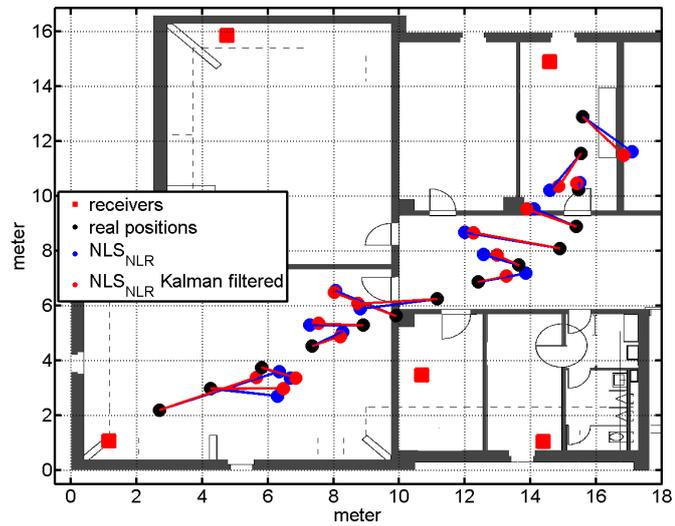


Gain) to the previous track. It can therefore eliminate certain peaks and reduce the influence of outlying measurements.

**Figure 4.14:** CDF of the Non-Linear Least Square with and without KF for NLR



**Figure 4.15:** Map of the Non-Linear Least Square Computed Locations for NLR



## 4.6 Performance of Extended Kalman Filter

To evaluate the performance of Extended Kalman Filter we compare the various localisation errors.

Log-normal Distance Path Loss

**Figure 4.16:** CDF of Extended Kalman Filter for LDPL

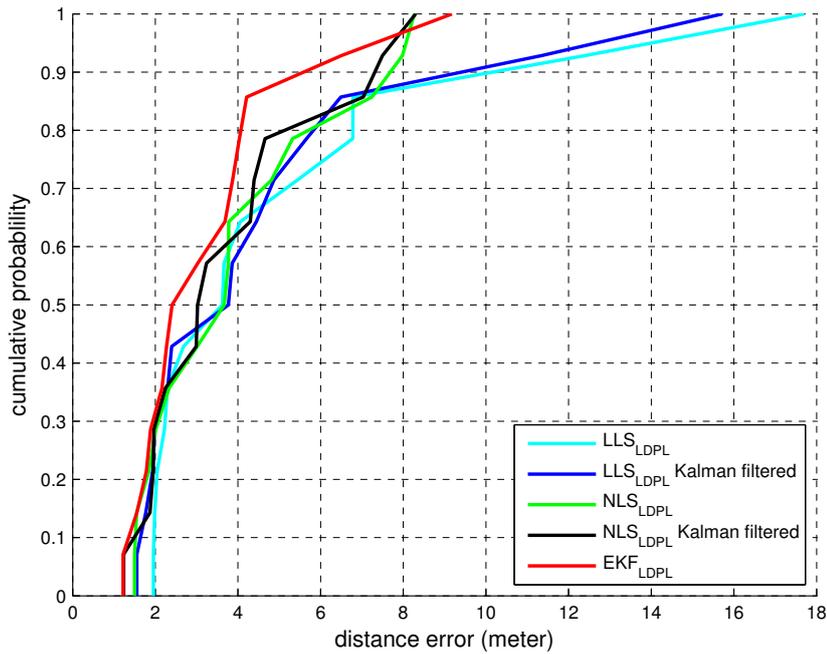


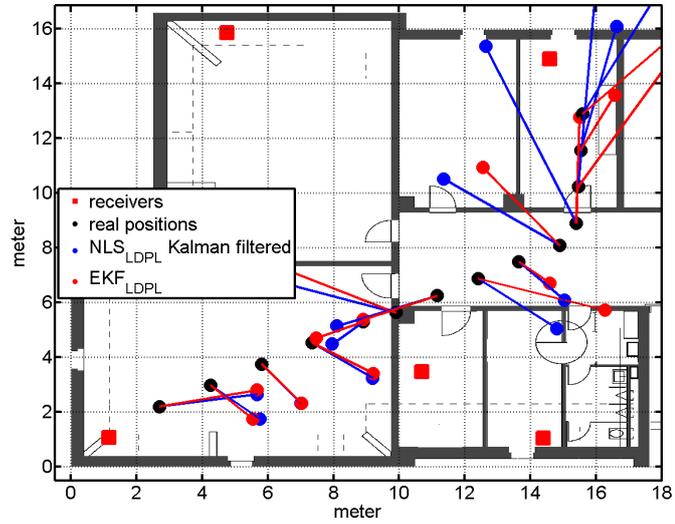
Figure 4.16 presents CDF of LLS, LLS with KF, NLS, NLS with KF and EKF using the LDPL model. Although the different CDFs start with different minimum errors they perform very similar up to 35% having errors up to 2.3 meters. At 80% the errors of EKF are smaller than 4 meters. Filtered NLS performs a bit worse with errors up to 5 meters at 80%. Although EKF has a maximum error of 9.1 meters it performs generally better than the NLS based localisations which have maximum errors of up to 8.2 meters.

Figure 4.17 shows the ground truth positions, the EKF positions and the filtered NLS positions.

### Non-linear Regression

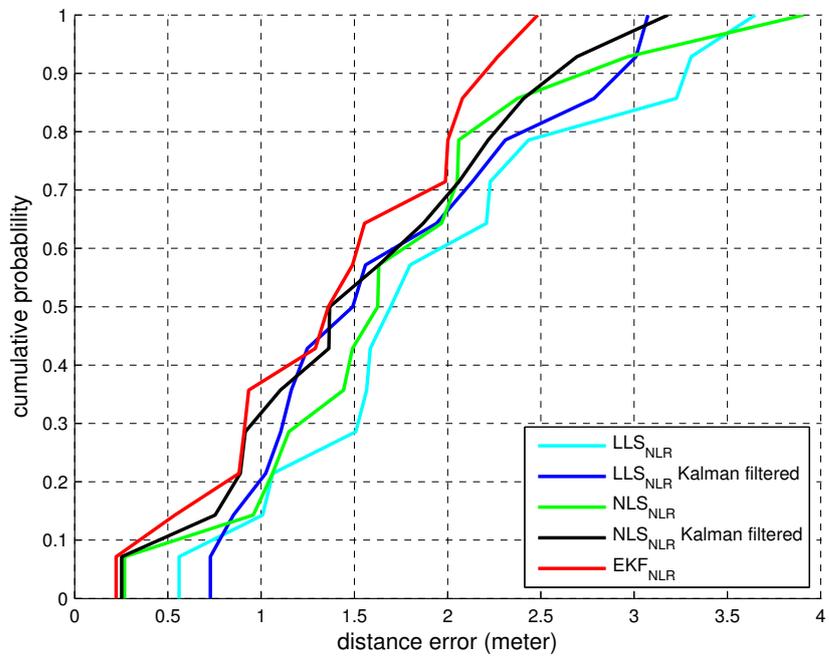
Figure 4.18 shows CDFs of LLS, LLS with KF, NLS, NLS with KF and EKF using the NLR model. Up to about 0.8% the curves for EKF and both NLS are almost the same. Up to 50% EKF is a bit better than the filtered NLS. At 50% both EKF and filtered NLS have an error of 1.4 meters. At 80% EKF and filtered NLS curve diverge and EKF is clearly the best with only

**Figure 4.17:** Map of Extended Kalman Filter Computed Locations for LDPL



a bit less than 2.5 meters as 100% error. Meanwhile the Kalman filtered NLS has 3.2 meters as 100% error.

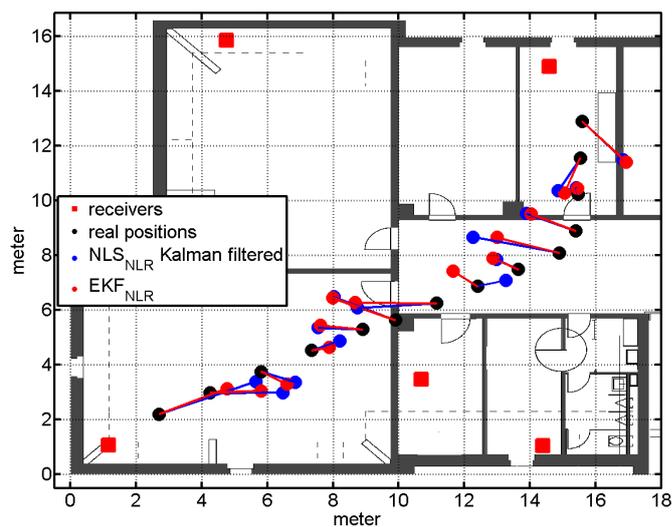
**Figure 4.18:** CDF of Extended Kalman Filter for NLR



EKF and the Kalman filtered results of NLS are very similar. This is because they have a lot of similarities. First both algorithms solve the same equation system (Equation 2.6). Second, both algorithms join the initial position estimation and the measurement information while considering their knowledge about the errors (state and measurement covariances). The Extended Kalman Filter has a main advantage over the other used localisation methods as it can directly use the distance information obtained from the Path Loss models instead of the two steps (localisation, filtering) used in the Least Squares approach. An other possible advantage of Extended Kalman Filter compared to Non-Linear Least Square is the way it solves the non-linear equation system. Instead of using recursive numerical computations EKF uses partial derivatives to determine the solution. Although this is not an advantage in accuracy it reduces the computational complexity significantly. Therefore, EKF can be used on low power devices such as sensor nodes.

For further comparison Figure 4.19 shows the map plot of the real positions and the EKF as well as the Kalman filtered NLS positions.

**Figure 4.19:** Map of Extended Kalman Filter Computed Locations for NLR





## Chapter 5

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# Conclusions

### 5.1 Summary and Conclusions

Indoor localisation is still not accurate enough to be considered as a solved problem in computer science. In this thesis we present a filter algorithm called Kalman Filter and its non-linear form the Extended Kalman Filter in Chapter 2. In the same chapter we present two different Path Loss models in indoor environments. We suggest to add a filtering step to the localisation to improve existing localisation algorithms such as Linear Least Square or Non-Linear Least Square. In Chapter 3 we introduce our localisation procedure and present a state model for our localisation on which we can apply Kalman Filter. In Chapter 4 we introduce a measurement setup to obtain data to evaluate our localisation system and filters. We further compare the two Path Loss models and use the better one to test our filter algorithms. We then evaluate the performance of Kalman Filter by comparing the localisation errors to the methods solely relying on trilateration algorithms, i.e., LLS and NLS. The evaluation shows that the localisation can be improved by applying Kalman Filter. As a conclusion of Chapter 4 we use the non-linear form of Kalman Filter (EKF) for localisation. We then compare the results to our previous localisations to the results of this non-linear Kalman Filter. The Extended Kalman Filter localisation is clearly superior to all other localisations we investigated in this thesis.

### 5.2 Future Work

Localisation based on RSS is heavily depending on accurate models of the signal attenuation in indoor environments. For good localisation results it is very important to get an accurate estimation of the distance between ANs and the target node. Therefore, more research should be done on how to improve our current PL models and developing more accurate models such as NLR. Also the amount of ANs and their placement is influencing the results. Further research could also be done in different filtering algorithms to improve existing localisations methods by adding one or more filter steps to the localisation procedure.



# List of Abbreviations

**AN** Anchor Node.

**CDF** Cumulative Distribution Function.

**dB** Decibel.

**EKF** Extended Kalman Filter.

**GPS** Global Positioning System.

**KF** Kalman Filter.

**LAN** Local Area Network.

**LDPL** Log-normal Distance Path Loss.

**LLS** Linear Least Square.

**LOS** Line Of Sight.

**MPP** Multi-Path Propagation.

**NLOS** Non Line Of Sight.

**NLR** Non-linear Regression.

**NLS** Non-Linear Least Square.

**PL** Path Loss.

**RSS** Received Signal Strength.

**SDR** Software Defined Radio.

**USRP** Universal Software Radio Peripheral.



## Bibliography

- [1] Sarkar, Tapan K., et al. "A survey of various propagation models for mobile communication", *Antennas and Propagation Magazine, IEEE* 45.3 (2003): 51-82.
- [2] R. Akl, D. Tummala, and X. Li, "Indoor propagation modeling at 2.4 ghz for ieee 802.11 networks", in *Wireless Networks and Emerging Technologies*, 2006.
- [3] V. Erceg, L. Greenstein, S. Tjandra, S. Parkoff, A. Gupta, B. Kulic, A. Julius, and R. Bianchi, "An empirically based path loss model for wireless channels in suburban environments", *Selected Areas in Communications, IEEE Journal on*, vol. 17, no. 7, pp. 1205–1211, Jul 1999.
- [4] Zan Li, Braun, T., Dimitrova, D.C. "A Passive WiFi Source Localization System based on Fine-grained Power-based Trilateration", *IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM)*, June 2015.
- [5] J. J. Caffery, "A new approach to the geometry of TOA location", in *Proc. IEEE Vehic. Technol. Conf. (VTC)*, vol. 4, Boston, MA, Sep. 2000, pp. 1943–1949.
- [6] S. Venkatesh and R. M. Buehrer, "A linear programming approach to NLOS error mitigation in sensor networks", in *Proc. IEEE Int. Symp. Information Processing in Sensor Networks (IPSN)*, Nashville, Tennessee, Apr. 2006, pp. 301–308.
- [7] Z. Li, W. Trappe, Y. Zhang, and B. Nath, "Robust statistical methods for securing wireless localization in sensor networks", in *Proc. IEEE Int. Symp. Information Processing in Sensor Networks (IPSN)*, Los Angeles, CA, Apr. 2005, pp. 91–98.
- [8] Sinan Gezici, Ismail Guven, and Zafer Sahinoglu. "On the Performance of Linear Least-Squares Estimation in Wireless Positioning Systems", 978-1-4244-2075-9, IEEE, 2008.
- [9] R. E. Kalman. "A New Approach to Linear Filtering and Prediction Problems". *Transaction of the ASME, Journal of Basic Engineering*, pages 35-45, 1960.
- [10] Greg Welch and Gary Bishop. "An Introduction to the Kalman Filter", TR 95-041, University of North Carolina at Chapel Hill, Chapel Hill, NC, USA, 2006.
- [11] Maria Isabel Ribeiro. "Kalman and Extended Kalman Filters: Concept, Derivation and Properties", Instituto Superior Técnico, Lisboa, Portugal, 2004.

- [12] Guvenc, Abdallah, Jordan and Dedeoglu. "*Enhancements to RSS Based Indoor Tracking Systems Using Kalman Filters*". University of New Mexico, 2003.
- [13] Yubin Zhao, Marcel Kyas. "*Comparing Centralized Kalman Filter Schemes for Indoor Positioning in Wireless Sensor Network*". Freie Universität Berlin, 2011.
- [14] Ali Shareef and Yifeng Zhu. "*Localization Using Extended Kalman Filters in Wireless Sensor Networks*". University of Maine, United States of America, 2009.
- [15] Z.Li, T. Braun, "*A Passive Source Localization System for IEEE 802.15.4 Signal*", University of Bern, 2015.