Searching for Backbones

A High-Performance Parallel Algorithm for Solving Combinatorial Optimization Problems

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Introduction to Physical Optimization Algorithms

Simulated Annealing

Metropolis Criterion:

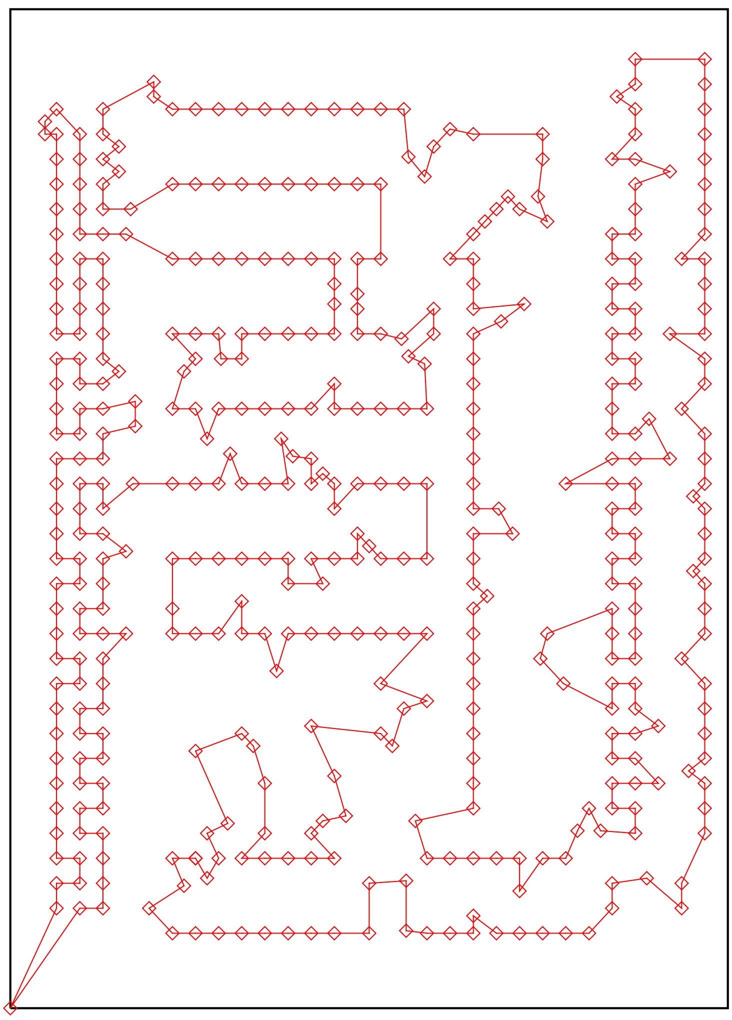
$$W(\sigma \rightarrow \tau) = \begin{cases} \exp\left(-\frac{\Delta\mathcal{H}}{k_B T}\right) & \text{if } \Delta\mathcal{H} \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

Threshold Accepting

$$W(\sigma \rightarrow \tau) = \begin{cases} 1 & \text{if } \Delta\mathcal{H} \leq \mathit{Th} \\ 0 & \text{otherwise} \end{cases}$$

Traveling Salesman Problem

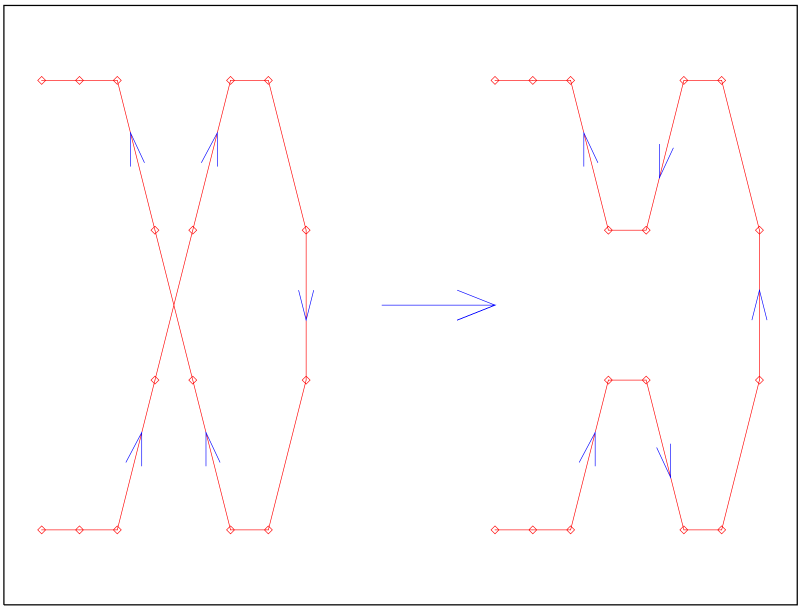
Famous benchmark: PCB442



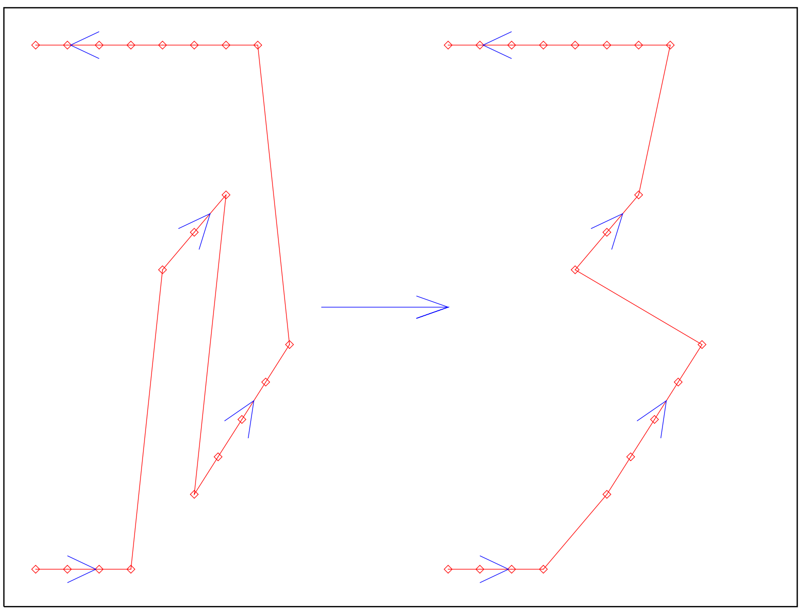
$$H(\sigma) = D(\sigma(N-1), \sigma(0)) + \sum_{i=0}^{N-2} D(\sigma(i), \sigma(i+1)) \quad (1)$$

Moves

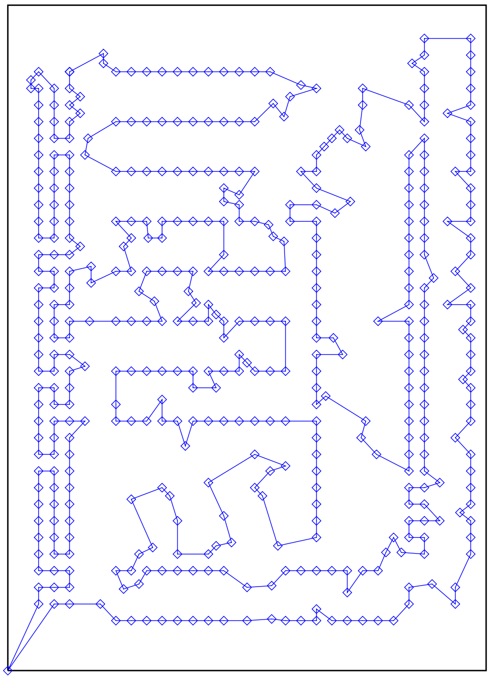
Lin-2-Opt

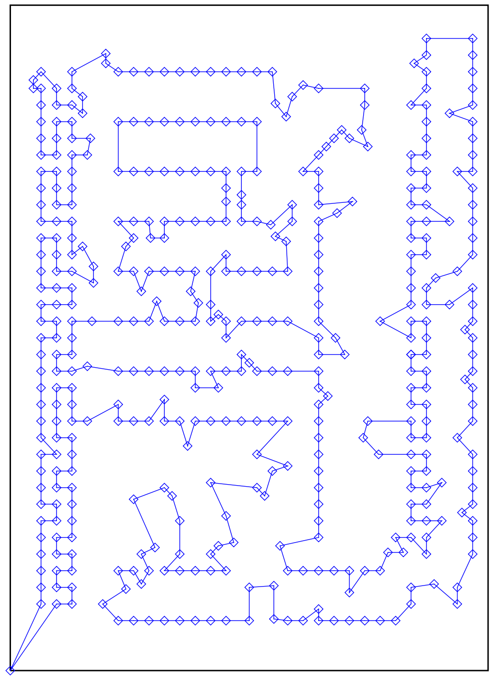


Lin-3-Opt



Searching for Backbones





Aim:

- Finding common structures in all solutions

- Reducing the complexity of the problem

- Saving calculation time and retrieving better results

Simple example:

4 solutions:

0	9	1	2	3	4	5	6	7	8
0	3	2	1	4	5	6	7	8	9
0	9	7	6	5	4	8	1	2	3
0	4	5	6	7	8	3	2	1	9

Backbone set:

0	9		
1	2	3	
4	5	6	7
8			

Coding:

$$\boxed{0 \quad 9} \Longrightarrow \boxed{0 \quad 1}$$
$$\boxed{1 \quad 2 \quad 3} \Longrightarrow \boxed{2 \quad 3}$$
$$\boxed{4 \quad 5 \quad 6 \quad 7} \Longrightarrow \boxed{4 \quad 5}$$
$$\boxed{8} \Longrightarrow \boxed{6}$$

New tour:

$$0-1 \quad 2-3 \quad 4-5 \quad 6-6$$

New 7×7 distance matrix:

$$\begin{aligned}
\widetilde{D}(0,1) &= D(0,9) \\
\widetilde{D}(2,3) &= D(1,2)+D(2,3) \\
\widetilde{D}(0,6) &= D(0,8)
\end{aligned}$$

New solutions:

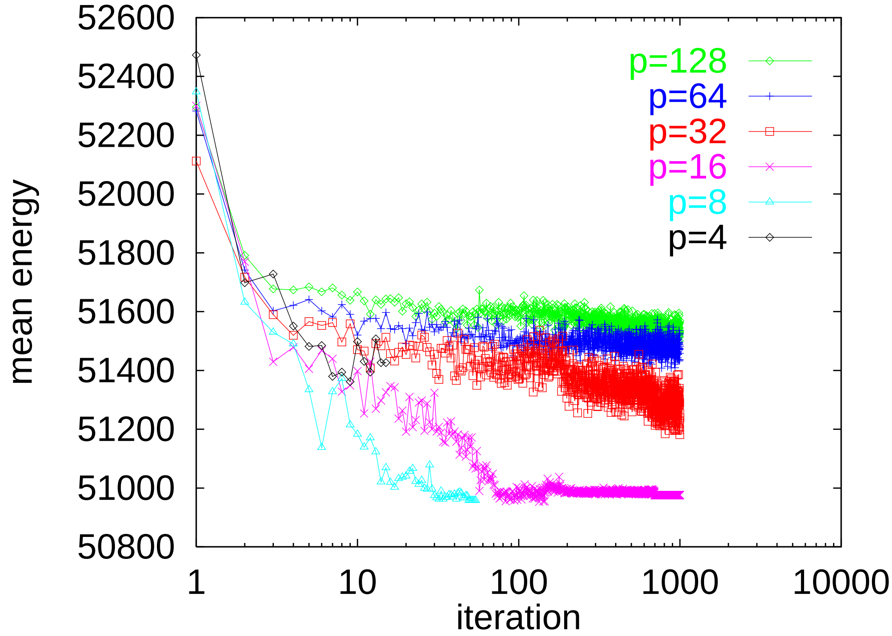
0 − 1	2 − 3	6 − 6	4 − 5
1 − 0	4 − 5	2 − 3	6 − 6
0 − 1	6 − 6	3 − 2	5 − 4
1 − 0	4 − 5	6 − 6	3 − 2

Decoded new solutions:

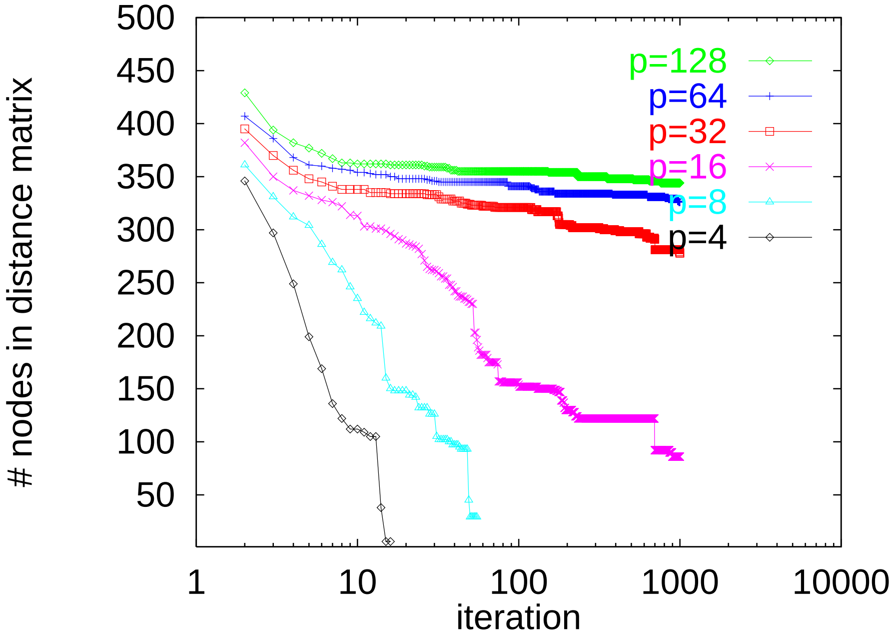
0	9	1	2	3	8	4	5	6	7
9	0	4	5	6	7	1	2	3	8
0	9	8	3	2	1	7	6	5	4
9	0	4	5	6	7	8	3	2	1

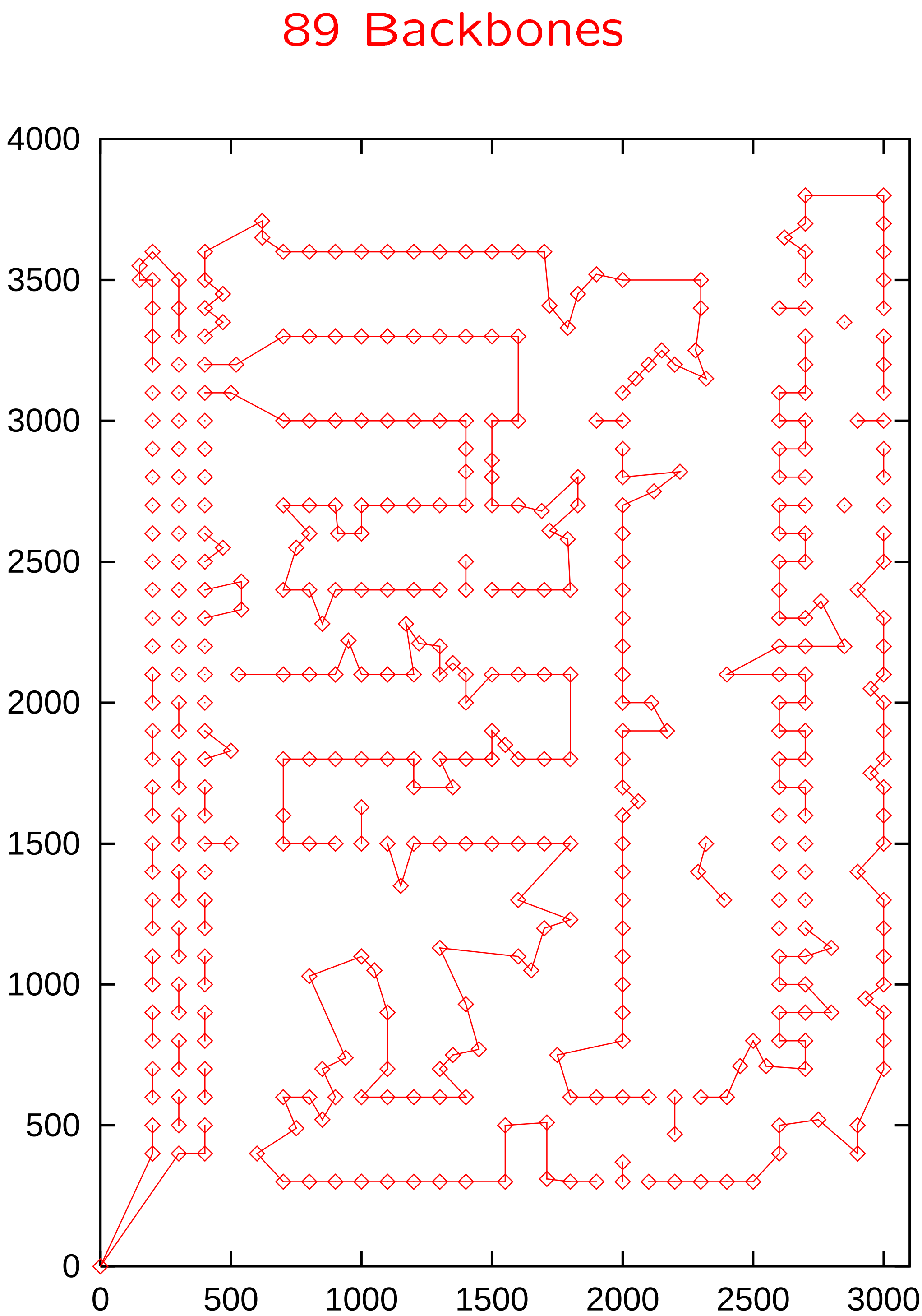
Results

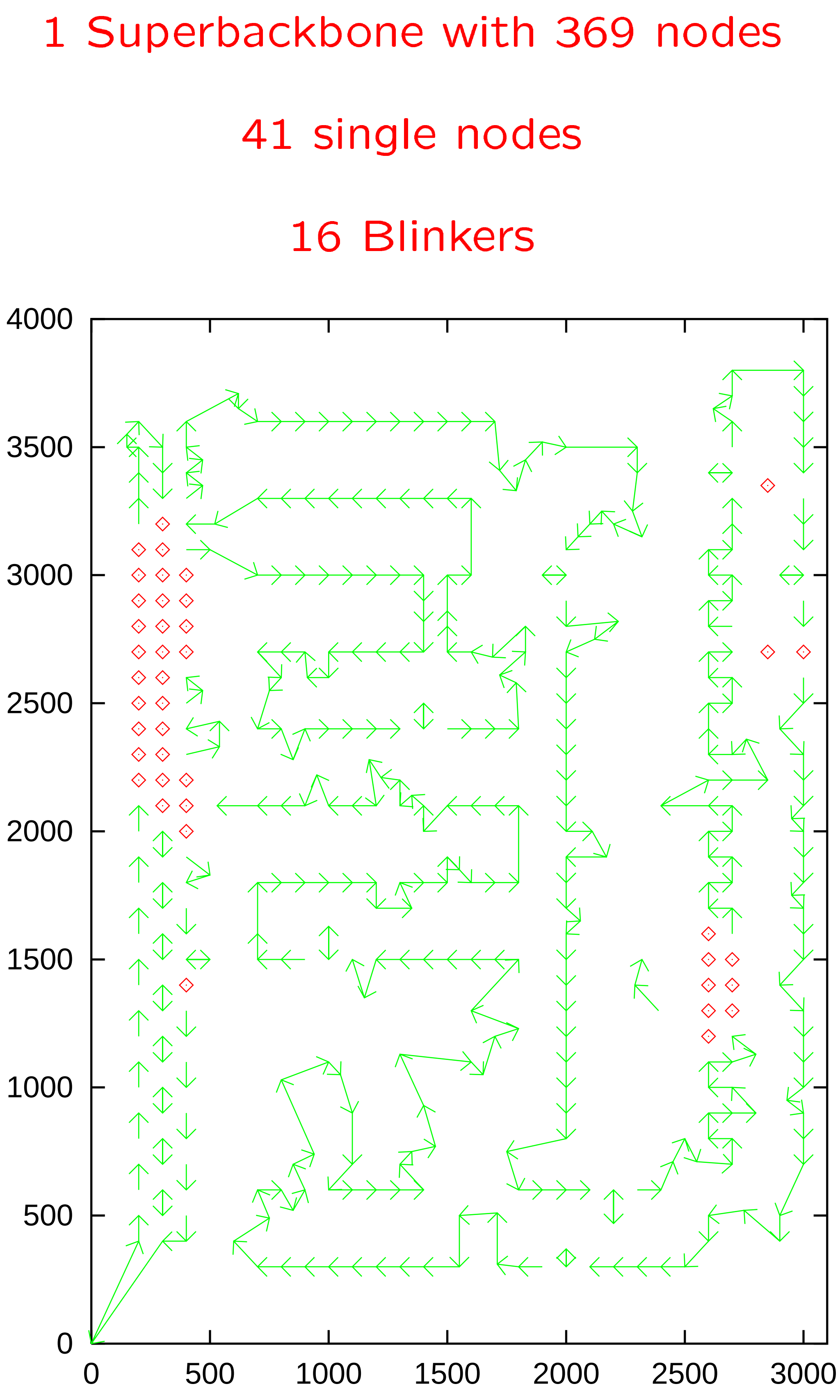
Improvement



Convergence




89 Backbones
0
500
1000
1500
2000
2500
3000
3500
4000
0
500
1000
1500
2000
2500
3000

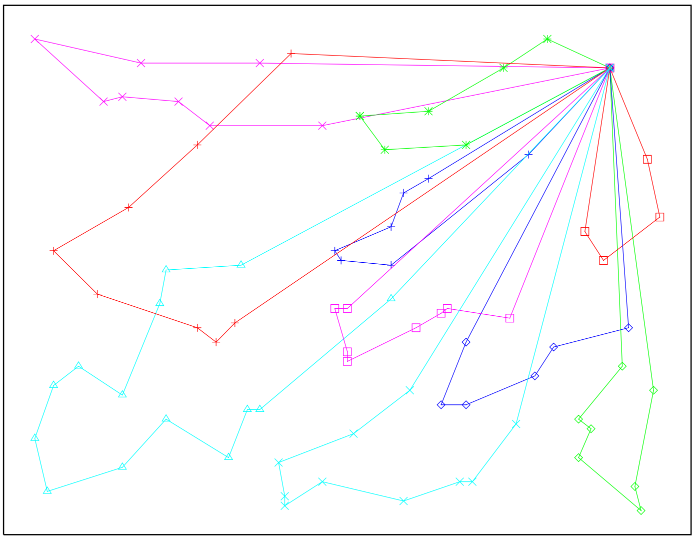

1 Superbackbone with 369 nodes
41 single nodes
16 Blinkers
4000
3500
3000
2500
2000
1500
1000
500
0
0
500
1000
1500
2000
2500
3000

Applications to other problems

Vehicle Routing Problem

TSP $\longrightarrow$ MTSP $\longrightarrow$ VRP

A-n80-k10



$$\mathcal{H}(\sigma) = \sum_{l=1}^{L} \sum_{i=1}^{N_l-1} D(\sigma(i,l),\sigma(i+1,l))$$

$$+ \lambda \sum_{l=1}^{L} \left(\sum_{i=2}^{N_l-1} m(\sigma(i,l)) - \kappa + \gamma \right)$$

$$\Theta \left(\sum_{i=2}^{N_l-1} m(\sigma(i,l)) - \kappa \right)$$